6–12 Find parametric equations and symmetric equations for the line.

7. The line through the points (1, 3, 2) and (−4, 3, 0)

$\bar{u} = \langle 1,3,2\rangle$, $\bar{v} = \langle -4,3,0\rangle$

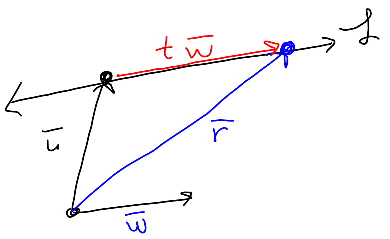
$\bar{u} - \bar{v} = \langle 5,0,2\rangle = \bar{w}$ is a vector in direction of the line.

Parametric equation:

Start with vector representation

$\boxed{\bar{r} = \bar{u} + t\bar{w}}$ is on the line.

$\bar{r}$ is position vector.



Straight to the point:

$\langle 1,3,2\rangle + t\langle 5,0,2\rangle$

$= \langle 5t+1, 3, 2t+2\rangle$

Parametric:

$x = 5t+1$, $y = 3$, $z = 2t+2$

Symmetric Equations

$\frac{x-1}{5} = \frac{z-2}{2}$, $y = 3$

10. The line through (2, 1, 0) and perpendicular to both **i** + **j** and **j** + **k**

$\boxed{\bar{u} = \langle 2,1,0\rangle}$

$\bar{u} = \bar{i} + \bar{j} = \langle 1,1,0\rangle$, $\bar{j} + \bar{k} = \langle 0,1,1\rangle = \bar{v}$

$\bar{u} \times \bar{v}$: $\langle 1,1,0\rangle, 1, 0$

$\langle 0,1,1\rangle, 0, 1$

$\bar{u} \times \bar{v} = \bar{w} = \boxed{\langle 1,1,1\rangle = \bar{w}}$

$\bar{r} = \langle 2,1,0\rangle + t\langle 1,1,1\rangle$

$\boxed{\bar{r} = \bar{u} + t\bar{w}}$ is vector eq'n for line.

Parametric

$\boxed{x = t+2,\ y = t+1,\ z = t}$

$\bar{r} = \langle t+2, t+1, t\rangle$

perfect, if you have $\bar{u}$ & $\bar{w}$ defined & boxed.

Symmetric (Bleah)

$(t=)$ $x-2 = y-1 = z$

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11. The line through $(1, -1, 1)$ and parallel to the line
$t = x + 2 = \frac{1}{2}y = z - 3$

Want direction vector $\bar{v}$ for the given line:

$x = t - 2$, $y = 2t$, $z = t + 3$

$\bar{v} = \langle 1, 2, 1 \rangle$

vector eq'n:

Let $\bar{u} = \langle 1, -1, 1 \rangle$ = position vector for $P(1, -1, 1) \in \mathcal{L}$

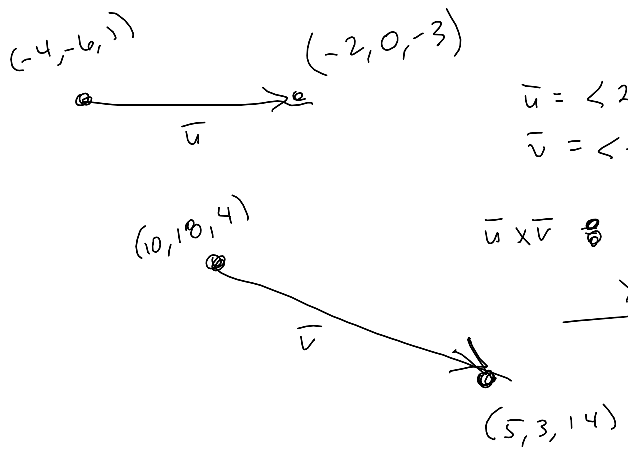
$\boxed{\bar{r} = \bar{u} + t\bar{v} \quad \forall t \in \mathbb{R}}$ vector

parametric: $\langle 1, -1, 1 \rangle + t\langle 1, 2, 1 \rangle$

$\boxed{x = t + 1, \; y = 2t - 1, \; z = t + 1}$

Symmetric: Bleah!

13. Is the line through $(-4, -6, 1)$ and $(-2, 0, -3)$ parallel to the line through $(10, 18, 4)$ and $(5, 3, 14)$?



$\bar{u} = \langle 2, 6, -4 \rangle$

$\bar{v} = \langle -5, -15, 10 \rangle$

$\bar{u} \times \bar{v} = \langle 2, 6, -4 \rangle \times \langle -5, -15, 10 \rangle$

$\langle 0, 0, 0 \rangle = \bar{0}$

Yes! because $\bar{u} \times \bar{v} = \bar{0}$

14. Is the line through $(4, 1, -1)$ and $(2, 5, 3)$ perpendicular to the line through $(-3, 2, 0)$ and $(5, 1, 4)$?

P = $(4, 1, -1)$, Q = $(2, 5, 3)$, R = $(-3, 2, 0)$, S = $(5, 1, 4)$

$\Rightarrow \overrightarrow{PQ} = \bar{u} = \langle -2, 4, 4 \rangle$

$\overrightarrow{RS} = \bar{v} = \langle 8, -1, 4 \rangle$

$\Rightarrow \bar{u} \cdot \bar{v} = \langle -2, 4, 4 \rangle \cdot \langle 8, -1, 4 \rangle$

$= -16 - 4 + 16 = 28 \neq 0$

$\bar{u} \cdot \bar{v} = 28 \neq 0 \Rightarrow$ Not $\perp$.

$\Rightarrow$ Not orthogonal

19–22 Determine whether the lines L_1 and L_2 are parallel, skew, or intersecting. If they intersect, find the point of intersection.

19. L_1: $x = -6t, \quad y = 1 + 9t, \quad z = -3t$

L_2: $x = 1 + 2s, \quad y = 4 - 3s, \quad z = s$

$x = -6t = 1 + 2s$

$y = 1 + 9t = 4 - 3s$

$z = -3t = s \Rightarrow y = 1 + 9t = 4 - 3(-3t) \Rightarrow$

$\Rightarrow 1 + 9t = 4 + 9t$

$\Rightarrow 1 = 4$ ✗ $\boxed{\text{No intersection!}}$

Absurd!

63. Find parametric equations for the line through the point (0, 1, 2) that is parallel to the plane $x + y + z = 2$ and perpendicular to the line $x = 1 + t$, $y = 1 - t$, $z = 2t$., i.e., $\langle x,y,z\rangle = \langle t+1, -t+1, 2t\rangle$

$\bar{v}$ = direction vector for the given line $\mathcal{L}$: $\langle 1,-1,2\rangle = \bar{v}$

Want a vector $\perp$ to this that is parallel the given plane $\mathcal{P}$.

I would prefer to stuff it all into a vector $\bar{u} = \langle u_1(t), u_2(t), u_3(t)\rangle$

$\parallel$ to $\mathcal{P}$ means $\perp$ to $\bar{n} = \langle 1,1,1\rangle$

Let $\bar{w}$ be direction vector for our line $\mathcal{L}_{63}$

$\bar{w} = \langle w_1, w_2, w_3\rangle$.

Then $\bar{n} \perp \bar{w} \Longrightarrow \bar{n}\circ\bar{w} = w_1 + w_2 + w_3 = 0$

$\perp$ to $\mathcal{L}$ $\quad \bar{v}\circ\bar{w} = 0 = w_1 - w_2 + 2w_3 = 0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 0 \end{array}\right] \xrightarrow{-R1+R2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 \end{array}\right] \sim \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \end{array}\right] \end{array}$$

$\Longrightarrow w_1 + w_2 + w_3 = 0$

$w_2 - \frac{1}{2}w_3 = 0 \Longrightarrow \boxed{w_2 = \frac{1}{2}w_3}$

$\Longrightarrow w_1 + \frac{1}{2}w_3 + w_3 = 0$

$\boxed{w_1 = -\frac{3}{2}w_3}$

$\{(w_1,w_2,w_3) \mid w_1 = -\frac{3}{2}w_3,\ w_2 = \frac{1}{2}w_3\}$

$= \{(x,y,z) \mid x = -\frac{3}{2}z,\ y = \frac{1}{2}z,\ z \in \mathbb{R}\}$

We'll pick a particular solution for $\bar{w}$.

$z = 2 \Longrightarrow x = -3,\ y = 1$

$\boxed{\bar{w} = \langle -3, 1, 2\rangle}$

Define $\boxed{\bar{r}_0 = \langle 0,1,2\rangle}$.

Then $\boxed{\bar{r} = \bar{r}_0 + t\bar{w}}$ describe the line in elegant fashion

$\bar{r} = \langle 0,1,2\rangle + t\langle -3,1,2\rangle$

Book wants:

$x = -3t,\ y = 1 + t,\ z = 2 + 2t$

62. (a) Find the point at which the given lines intersect:

$$\mathbf{r}_1 = \langle 1, 1, 0\rangle + t\langle 1, -1, 2\rangle$$
$$\mathbf{r}_2 = \langle 2, 0, 2\rangle + s\langle -1, 1, 0\rangle$$

(b) Find an equation of the plane that contains these lines.

$\bar{r}_1 = \langle 1+t, 1-t, 0+2t\rangle$

$\bar{r}_2 = \langle 2-s, 0+s, 2\rangle$

(a) $\bar{r}_1 = \bar{r}_2$:

$1+t = 2-s$

$1-t = -s$

$2t = 2 \Rightarrow \boxed{t = 1}$

$1-1 = \boxed{-s = 0}$

$t = 1, s = 0 \Rightarrow$

$\bar{r}_1 = \langle 1+1, 1-1, 0+2\rangle = \langle 2, 0, 2\rangle$

$\bar{r}_2 = \boxed{\langle 2, 0, 2\rangle \equiv \bar{r}_0}$

(b) $\bar{r}_1 = \bar{a} + t\bar{v}$

$\bar{r}_2 = \bar{b} + s\bar{w}$

Find $\bar{n}$: $\bar{v} \times \bar{w} = \bar{n}$

$\langle 1, -1, 2\rangle, 1, -1$

$\times\ \langle -1, 1, 0\rangle, -1, 1$

$\boxed{\langle -2, -2, 0\rangle = \bar{n}}$

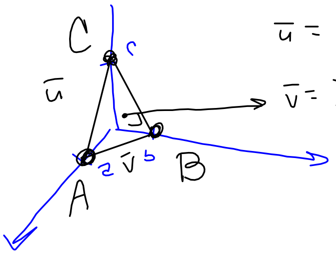
$\Rightarrow -2(x-2) + -2(y-0) + 0(z-2) = 0$

$\Rightarrow -2(x-2) - 2y = 0$

61. Find an equation of the plane with x-intercept a, y-intercept b, and z-intercept c.

$A(a,0,0)$ $B(0,b,0)$ $C(0,0,c)$

3 points: $\langle a, 0, 0\rangle, \langle 0, b, 0\rangle, \langle 0, 0, c\rangle$



$\bar{u} = \overrightarrow{AC} = \langle -a, 0, c\rangle$

$\bar{v} = \overrightarrow{AB} = \langle -a, b, 0\rangle$

$\bar{u} \times \bar{v}$ $\langle -a, 0, c\rangle, -a, 0$

$\times\ \langle -a, b, 0\rangle, -a, b$

$\boxed{\langle -bc, -ac, -ab\rangle = \bar{n}}$

Use A:

$\boxed{-bc(x-a) - ac(y-0) - ab(z-0) = 0}$

From $\bar{n} \cdot \langle x-a, y, z\rangle = 0$

Jan 28-7:25 PM