Question asked in class:

44. (a) Let P be a point not on the plane that passes through the points Q, R, and S. Show that the distance d from P to the plane is

$$d = \frac{|(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}|}{|\mathbf{a} \times \mathbf{b}|}$$

where $\mathbf{a} = \overrightarrow{QR}$, $\mathbf{b} = \overrightarrow{QS}$, and $\mathbf{c} = \overrightarrow{QP}$.

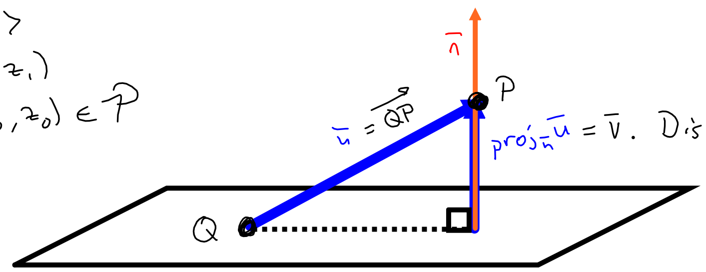
Let's develop the theory, starting with the distance from a point to a plane. The distance between two planes is a spin-off.

$\mathcal{P}$: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$\bar{n} = \langle a,b,c \rangle$

$P = (x_1,y_1,z_1)$

$Q = (x_0,y_0,z_0) \in \mathcal{P}$



$\text{proj}_{\bar{n}}\bar{u} = \bar{v}$. Distance $= D = \|\bar{v}\|$

$= |\text{comp}_{\bar{n}}\bar{u}|$

$\text{comp}_{\bar{n}}\bar{u} = \dfrac{\bar{u}\cdot\bar{n}}{\|\bar{n}\|}$, where $\bar{u} = \overrightarrow{QP}$

$\bar{u} = \overrightarrow{QP} = \langle x_1-x_0, y_1-y_0, z_1-z_0\rangle \Rightarrow \bar{u}\cdot\bar{n} = a(x_1-x_0)+b(y_1-y_0)+c(z_1-z_0)$

$$\Rightarrow |\text{comp}_{\bar{n}}\bar{u}| = \left|\frac{a(x_1-x_0)+b(y_1-y_0)+c(z_1-z_0)}{\sqrt{a^2+b^2+c^2}}\right| = D$$

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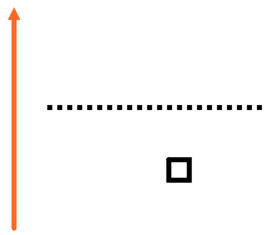
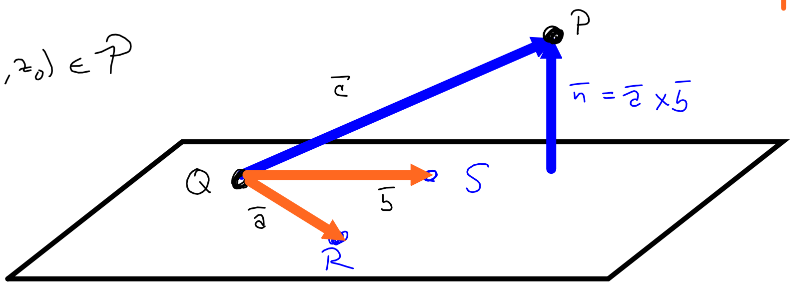
$\bar{u} = \overrightarrow{QP}$



$= (x_0, y_0, z_0) \in \mathcal{P}$



$\text{proj}_{\bar{n}} \bar{u} = \bar{v}$. Distance $= |\text{proj}_{\bar{n}} \bar{u}|$

$$D = \left| \frac{\bar{c} \cdot \bar{n}}{\|\bar{n}\|} \right| = \left| \frac{\bar{c} \cdot (\bar{a} \times \bar{b})}{\|\bar{a} \times \bar{b}\|} \right| = \left| \frac{(\bar{a} \times \bar{b}) \cdot \bar{c}}{\|\bar{a} \times \bar{b}\|} \right|$$ b/c is commutative.