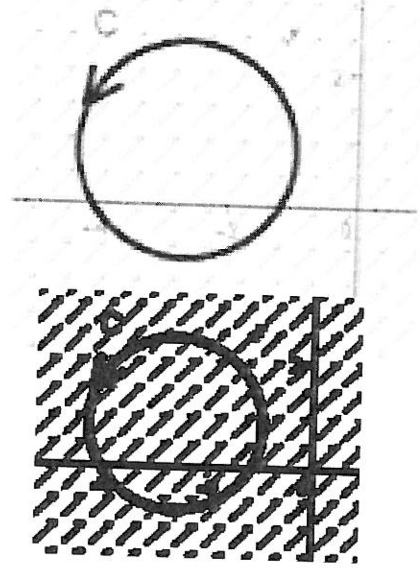
MAT 203-G11
Covers Chapter 16

Final Test – Spring, 2016

Name KEY

Take this test like a normal test, but do not staple your work, before turning it in. You have until 1:50 p.m. to finish what you can, in one sitting. Unless anyone objects, I'd like you to take the test home and finish it, by noon on Thursday. If class votes against Take-home, we'll go clear to 2:00 p.m.

1. (10 pts) The plot of a field and a smooth, closed, oriented curve C are shown. Would you estimate that $\int_C \overline{F} \bullet d\overline{r}$ is negative, zero, or positive?



The two images are the same field for #1. Just wanted to help us see it.

2. (10 pts) Evaluate the line integral $\int_C x\,ds$, where S is the line segment joining $(-2,-3)$ to $(3,2)$.

3. Let $\overline{F}(x,y,z)=\langle y^2, 2xy+e^{3z}, 3ye^{3z}\rangle = y^2\mathbf{i}+(2xy+3e^{3z})\mathbf{j}+3ye^{3z}\mathbf{k}$.

   a. (10 pts) Show that $\overline{F}$ is a conservative field.

   b. (10 pts) Find a potential function f such that $\overline{F}=\nabla f$.

4. Set up the integral $\int_C x\,dy$ for evaluation, where C is the triangle with vertices $(0,0),(0,4)$, and $(3,0)$ in two ways:

   a. (10 pts) Directly, as the sum of 3 line integrals along the edges of C. (Write the 3 integrals, but do not evaluate.)

   b. (10 pts) Using Green's Theorem. (Write the iterated integral, but do not evaluate.).
   I think the above problem is do-able, but maybe time-consuming. Beware the clock on this one.

5. Let $\overline{F}(x,y,z)=\langle xye^z, xze^y, yze^x\rangle$.

   a. (5 pts) Find the divergence of $\overline{F}$.

   b. (5 pts) Find the curl of $\overline{F}$.

6. Express the area of the part of the surface $z=5-x^2-y^2$ that lies within the cylinder $x^2+y^2=1$, as an iterated integral, in …

   a. (5 pts) … rectangular coordinates. (Do not evaluate.)

   b. (5 pts) … polar coordinates. (Do not evaluate.)

   Take-Home HINT: $\iint_D dS \approx 5.330413500$

7. (10 pts) Find an equation, in rectangular coordinates, for the tangent plane to the surface $\bar{r}(u,v)=\langle v,u^2+v^2,u\rangle$ at the point $(1,1,0)$. In other words, I'm looking for something of the form $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$ as your final answer.

8. Suppose $\bar{F}(x,y,z)=\langle 2xye^z,-y^2e^z,z\rangle$ and E is the intersection of the solid ball $x^2+y^2+z^2\le 1$ with the first octant ($z\ge 0$). Use the Divergence Theorem to express the flux of $\bar{F}$ across the surface S in two ways:

   a. (5 pts) $\iint_S \bar{F}\bullet d\bar{S}$, which is written in your book as $\iint_S \mathbf{F}\bullet d\mathbf{S}$

   b. (5 pts) $\iiint_E div\,\mathbf{F}\,dV$

   I want you to take them to the iterated integrals, for full credit. Keep in mind there's some bonus, below, too. If you hit a snag on this one, I suggest moving on to easier points! Then come back.

Bonus

9. Write the iterated integral for the triple integral $\iiint_E G(x,y,z)dV$, where the solid E is bounded by the paraboloid $z=4-x^2-y^2$, the cylinder $y=|x|$, and the plane $z=0$. All we know is that $G(x,y,z)$ is proportional to its distance from the z-axis, which is acting a bit like a charged wire. Write 2 iterated integrals for this triple integral:

   a. (5 pts) One in rectangular coordinates.

   b. (5 pts) One in cylindrical coordinates.

10. (5 pts) Find the Jacobian for the transformations

   a. $u=x+2y$, $v=2x+3y$ Take your time, be careful with your fractions and you'll get done quicker.

11. (10 pts) Find $\frac{\partial f}{\partial x}=f_x$ and $\frac{\partial f}{\partial y}=f_y$ for $f(x,y)=\int_{3y^2-2y}^{\sin(x)\cos(x)} \frac{\cos(t)}{\sin(t)+5}\,dt$. FTC I with Chain Rule!

203 FINAL SPR '16

(1) $\int_C \bar{F} \cdot d\bar{r} \; \boxed{=0}$

Closed loop. Uniform field.

$\bar{F}$ appears to be $c\langle 1,1\rangle$ for some $c>0$. $Q_x = P_y = 0$

10 pts

(2) $\int_C x\,ds$, C from $(-2,-3)$ to $(3,2)$

Think $s(t) = \bar{r}(t)$

$\frac{ds}{dt} = s'(t) = \bar{r}'(t)$

$ds = s'(t)\,dt = \bar{r}'(t)\,dt$

$\bar{r}(t) = (1-t)\langle -2,-3\rangle + t\langle 3,2\rangle$

$= \langle -2+2t, -3+3t\rangle + \langle 3t, 2t\rangle$

$= \langle -2+5t, -3+5t\rangle$

$\boxed{x = 5t-2}$

10 pts

$\bar{r}'(t) = \langle 5,5\rangle$

$\|\bar{r}'(t)\| = \sqrt{5^2+5^2}$

$= \sqrt{2\cdot 25}$

$= 5\sqrt{2}$

$ds = \sqrt{x_t^2 + y_t^2}\,dt = \sqrt{5^2+5^2}\,dt = 5\sqrt{2}\,dt$

$\Longrightarrow \int_C x\,ds = \int_0^1 (5t-2)\,5\sqrt{2}\,dt$

$= 5\sqrt{2}\left[\frac{5}{2}t^2 - 2t\right]_0^1 = 5\sqrt{2}\left[\frac{5}{2} - 2\right]$

$= 5\sqrt{2}\left[\frac{5-4}{2}\right] = \boxed{\frac{5}{2}\sqrt{2}}$

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(3) $\bar{F} = \langle y^2, 2xy + e^{3z}, 3ye^{3z} \rangle$ $\bar{F}$ is continuously differentiable $\forall \bar{x} \in \mathbb{R}^3$

(a) $\text{curl } \bar{F} = \nabla \times \bar{F}$:

$$\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$$
$$\times \langle y^2, 2xy + e^{3z}, 3ye^{3z} \rangle$$

$$\langle 3e^{3z} - 3e^{3z}, -(0-0), 2y - 2y \rangle$$

$= \bar{0} \Longrightarrow$ Conservative.

10pts

(b) $f_x = y^2 \longrightarrow f = xy^2 + \alpha(y,z)$

10pts

$\Longrightarrow f_y = 2xy + \alpha_y(y,z) = 2xy + e^{3z}$

So $\alpha_y(y,z) = e^{3z} \longrightarrow$

$\alpha(y,z) = ye^{3z} + \beta(z)$

So $f = xy^2 + ye^{3z} + \beta(z)$

& $f_z = 3ye^{3z} + \beta'(z) = 3ye^{3z} \longrightarrow \beta'(z) = 0$

So, $\boxed{f = xy^2 + ye^{3z} + C}$ $\forall C \in \mathbb{R}$.

$\frac{1}{3}(3ye^{3z})$

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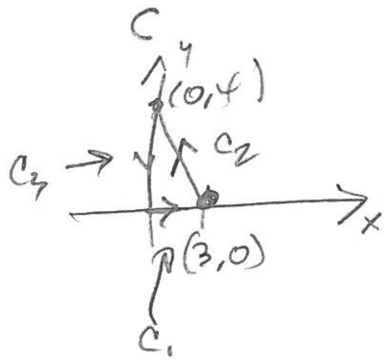
(4) $\int x\,dy$. C is triangle: (0,0), (0,4), (3,0)



2 ways:

(a) Line

(b) Green's

10 pts

C_1: $(1-t)(0,0) + t\langle 3,0\rangle = \langle 3t, 0\rangle = \vec{r}$

But $dy \equiv 0$ on this set, so

$$\boxed{\int_{C_1} = 0}$$

C_2: $(1-t)\langle 3,0\rangle + t\langle 0,4\rangle$

$= \langle 3-3t, 4t\rangle = \vec{r}$

$x_t = -3,\ y_t = 4$

$x = 3-3t \quad y = 4t$

$dy = 4\,dt$

$$\int_{C_2} x\,dy = 4\int_0^1 (3-3t)\,dt = 4\left[3t - \frac{3}{2}t^2\right]_0^1 = 4\left[3 - \frac{3}{2}\right] = 4\left(\frac{3}{2}\right) = \boxed{6}$$

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C_3 : $x \equiv 0$, so $\int_{C_3} x\,dy = 0$.

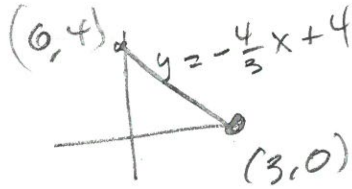
$\sum \int_{C_k} = 0 + 6 + 0 = 6 = \int_C x\,dy$

(b) Green's $\int_C P\,dx + Q\,dy = \iint_S (Q_x - P_y)\,dA$

10 pts

$Q_x = 1$, $P_y = 0$

So, $\iint_S 1\,dA$



$\bar{r} = \langle$

$\int_0^3 \int_0^{-\frac{4}{3}x+4} dy\,dx$

TYPE I

$= \int_0^3 \left[y \right]_0^{-\frac{4}{3}x+4} dx = \int_0^3 \left(-\frac{4}{3}x + 4\right) dx$

$= \left[-\frac{4}{6}x^2 + 4x \right]_0^3 = \left(-\frac{4}{6}\right)(9) + 4(3) = \left(-\frac{2}{3}\right)(9) + 12$

$= -6 + 12 = \boxed{6}$ ✓ GOOD!

203 FINAL

5 $\bar{F} = \langle xye^{z}, xze^{y}, yze^{x} \rangle$

a $\operatorname{div} \bar{F}$: b $\operatorname{curl} \bar{F}$

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a $\operatorname{div} \bar{F} = \nabla \cdot \bar{F} = \boxed{ye^{z} + xze^{y} + ye^{x}}$

5pts

b $\operatorname{curl} \bar{F} = \nabla \times \bar{F}$;

$\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle$

$\langle xye^{z}, xze^{y}, yze^{x} \rangle$

$\langle ze^{x} - xe^{y}, -(yze^{x} - xye^{z}), ze^{y} - xe^{z} \rangle$

$= \boxed{\nabla \times \bar{F} = \langle ze^{x} - xe^{y}, xye^{z} - yze^{x}, ze^{y} - xe^{z} \rangle}$

5pts

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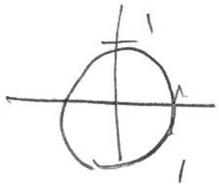
(6) $z = 5 - x^2 - y^2$ inside $x^2 + y^2 = 1$

Surface area ...

(a) Rectangular: $\boxed{\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{4x^2+4y^2+1} \, dy \, dx}$



$dS = \|\bar{r}_x \times \bar{r}_y\| \, dy \, dx$

5pts

$\bar{r} = \langle x, y, 5 - x^2 - y^2 \rangle$

$\bar{r}_x = \langle 1, 0, -2x \rangle$

$\times \quad \bar{r}_y = \langle 0, 1, -2y \rangle$

$\langle 2x, 2y, 1 \rangle \implies$

$\|\bar{r}_x \times \bar{r}_y\| = \sqrt{4x^2 + 4y^2 + 1}$

(b) $\boxed{\int_0^{2\pi} \int_0^1 \sqrt{(4r^2+1)} \, r \, dr \, d\theta}$

5pts

$\sqrt{4(x^2+y^2)+1} = \sqrt{4r^2+1}$

203 FINAL #6

This one is especially tricky, because going to polars involves an $f(x,y)$ going from dS to $\|\bar{r}_x \times \bar{r}_y\| \, dA$ =

$$= f(x,y)\, dA = \sqrt{4x^2+4y^2+1}\, dA$$

to $f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$

$$= \text{"}\sqrt{4x^2+4y^2+1}\text{"}\, r\,dr\,d\theta$$

$$= \sqrt{4r^2\cos^2\theta + 4r^2\sin^2\theta + 1}\; r\,dr\,d\theta$$

$$= \sqrt{4r^2(\cos^2\theta + \sin^2\theta) + 1}\; r\,dr\,d\theta$$

$= \sqrt{4r^2+1}\; r\,dr\,d\theta$, which is actually do-able, by hand!

$u = 4r^2+1 \Rightarrow du = 8r\,dr$

So $8r\,dr = du$

$\Rightarrow r\,dr = \frac{du}{8}$!

203 Fin #6 cnt'd

This gives

$$\iint_S dS = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \|\bar{r}_x \times \bar{r}_y\| \, dA$$

$$= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{4x^2+4y^2+1}\, dy\, dx$$

$$= \int_0^{2\pi}\int_0^1 \sqrt{4r^2+1}\; r\, dr\, d\theta$$

$$= \int_0^{2\pi} d\theta \int_0^1 \sqrt{4r^2+1}\; r\, dr = \int_0^{2\pi} d\theta \int_{r=0}^{r=1} \sqrt{u}\, \frac{du}{8}$$

$$= 2\pi \cdot \frac{1}{8}\int_{r=0}^{r=1} u^{\frac{1}{2}}\, du = \frac{\pi}{4}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_{r=0}^{r=1}$$

$$= \frac{\pi}{6}\left(4r^2+1\right)^{\frac{3}{2}}\Big]_0^1 = \frac{\pi}{6}\left(5^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) = \boxed{\frac{\pi}{6}\left(5\sqrt{5}-1\right)}$$

203 FINAL

7 $\bar{r} = \langle v, u^2+v^2, u \rangle$, $\bar{x}_0 = \langle 1, 1, 0 \rangle$

10 pts

Tangent Plane : $\Rightarrow (u,v) = (0,1)$

$x = 1 = v$

$y = u^2 + v^2$

$= u^2 + 1^2 = 1$

$\Rightarrow u = 0.$

$\bar{r}_u = \langle 0, 2u, 1 \rangle$

$\times \quad \bar{r}_v = \langle 1, 2v, 0 \rangle$

$\langle -2v, 1, -2u \rangle = \bar{n}(u,v)$

@ $(u,v) = (0,1)$:

$\langle -2, 1, 0 \rangle = \bar{n}.$

Now $\bar{x} \in P \Rightarrow \bar{n} \cdot (\bar{x} - \bar{x}_0) = 0$:

$$\boxed{-2(x-1) + (y-1) + 0(z-0) = 0}$$

Better :

$-2(x-1) + 1(y-1) = 0$

Variations

$-2x + 2 + y - 1 = 0$

$-2x + y = -1$

$2x - y = 1$

203 FINAL

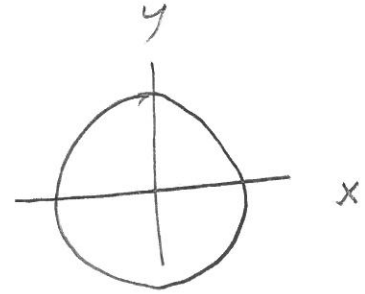
8) $\bar{F} = \langle 2xye^z, -y^2e^z, z \rangle$

S is sphere $x^2+y^2+z^2=1$

we find flux across S in 2 ways:

a) Directly $\iint_S \bar{F} \cdot d\bar{S}$:

S_2 :



$\bar{r}$: $\langle x, y, \sqrt{1-x^2-y^2} \rangle$

$\bar{r}_x$ $\langle 1, 0, \frac{-x}{\sqrt{1-x^2-y^2}} \rangle$

$\times$ $\bar{r}_y$ $\langle 0, 1, \frac{-y}{\sqrt{1-x^2-y^2}} \rangle$

$\langle \frac{x}{\sqrt{1-x^2-y^2}}, \frac{y}{\sqrt{1-x^2-y^2}}, 1 \rangle$. This gives us all we need:

$$\iint_{S_2} \bar{F} \cdot d\bar{S} = \iint_{S_2} \bar{F} \cdot \bar{n}\, dS = \iint_{S_2} \bar{F} \cdot (\bar{r}_x \times \bar{r}_y)\, dA$$

$$\iint_{S_1} \bar{F} \cdot \bar{n}\, dS + \iint_{S_2} \bar{F} \cdot d\bar{S}$$

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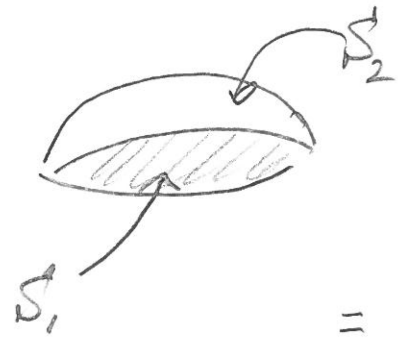
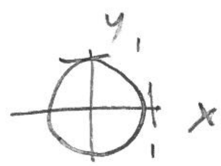
S_1: $x^2+y^2 \le 1$, $\bar{n} = \langle 0,0,-1\rangle$



$\iint_{S_1} \bar{F}\cdot\bar{n}\,dS$

$= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 0\,dS = 0$



$\bar{F}\cdot\bar{n} = \langle 2xye^z, -y^2e^z, z\rangle \cdot \langle 0,0,-1\rangle$

$= -z$ & $z\equiv 0 \Longrightarrow \iint_{S_1} = 0$

S_2; $z = \sqrt{1-x^2-y^2}$

$\bar{r} = \langle x, y, \sqrt{1-x^2-y^2}\rangle$

$\iint_{S_2} \bar{F}\cdot d\bar{S} = \iint_{S_2} \langle 2xye^z, -y^2e^z, z\rangle \cdot \left\langle \frac{x}{\sqrt{1-x^2-y^2}}, \frac{y}{\sqrt{1-x^2-y^2}}, 1\right\rangle dA$

$= \iint_{S_2} \left(\frac{2x^2ye^z}{\sqrt{1-x^2-y^2}} - \frac{y^3e^z}{\sqrt{1-x^2-y^2}} + z\right) dA$

$= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \left(\frac{2x^2ye^{\sqrt{1-x^2-y^2}}}{\sqrt{1-x^2-y^2}} - \frac{y^3e^{\sqrt{1-x^2-y^2}}}{\sqrt{1-x^2-y^2}} + \sqrt{1-x^2-y^2}\right) \cdot dy\,dx.$

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$$= \int_0^{2\pi}\int_0^1 \left(\frac{2r^3\cos^2\theta\sin\theta\, e^{\sqrt{1-r^2}}}{\sqrt{1-r^2}} - \frac{r^3\sin^3\theta\, e^{\sqrt{1-r^2}}}{\sqrt{1-r^2}} + \sqrt{1-r^2} \right) \cdot r\,dr\,d\theta = \frac{2\pi}{3}!$$

by Wolfram Alpha

(b) $\iiint_E \operatorname{div}\bar{F}\,dV$

$$\bar{F} = \langle 2xye^z, -y^2e^z, z \rangle$$

$$\operatorname{div}\bar{F} = 2ye^z - 2ye^z + 1$$

$$\iiint_E 1\,dV = \int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^1 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

This is do-able, by hand.

$$= \left[\int_0^{2\pi} d\theta \int_0^{\frac{\pi}{2}} \sin\phi\,d\phi\right]\left[\frac{1}{3}\rho^3\right]_0^1 =$$

$$= (2\pi)(1)\left(\frac{1}{3}\right) = \boxed{\frac{2\pi}{3}}$$

Rectangular $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} dz\,dy\,dx$.

203 Turning crank on 8a

$$= \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 2x^2 y e^z \text{, etc}$$

$$= \int_{0}^{2\pi} 2\cos^2\theta \sin\theta \, d\theta$$

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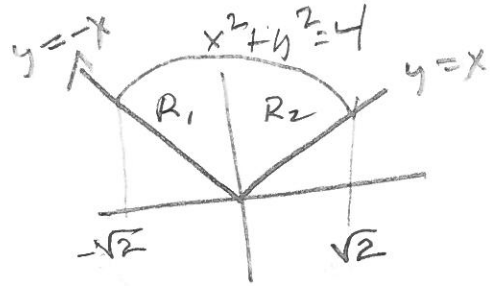
9 $z = 4-x^2-y^2$ Lid.

sides $\begin{cases} y = |x| \text{ sidewalls} \\ z = 0 \end{cases}$



$$\iiint_E G(x,y,z)\,dV = \int_{-\sqrt{2}}^{0}\int_{-x}^{\sqrt{4-x^2}}\int_0^{4-x^2-y^2} K\sqrt{x^2+y^2}\,dz\,dy\,dx$$

a 5 pts

$$+\int_0^{\sqrt{2}}\int_{x}^{\sqrt{4-x^2}}\int_0^{4-x^2-y^2} K\sqrt{x^2+y^2}\,dz\,dy\,dx$$

$$=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\int_0^2\int_0^{4-r^2} K\,r\cdot r\,dz\,dr\,d\theta$$

b 5 pts

$$=K\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\theta\int_0^2\left[r^2 z\right]_0^{4-r^2} dr\,d\theta = \left[K\left(\frac{3\pi}{4}-\frac{\pi}{4}\right)\right]\cdot\int_0^2 (4r^2-r^4)\,dr$$

$$=\frac{K\pi}{2}\left[\frac{4}{3}r^3-\frac{1}{5}r^5\right]_0^2 = \frac{K\pi}{2}\left[\frac{32}{3}-\frac{32}{5}\right] = \frac{K\pi}{2}\left[\frac{64}{15}\right]$$

$$=\boxed{\frac{32K\pi}{15}}$$

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10 5pts

$u = x + 2y, \quad v = 2x + 3y$

So $x = u - 2y \Rightarrow v = 2(u - 2y) + 3y$

$= 2u - 4y + 3y$

$= 2u - y$

$\rightarrow \boxed{y = 2u - v}$

$\Rightarrow x = u - 2(2u - v)$

$= u - 4u + 2v$

$= -3u + 2v$

$\bar{r} = \langle -3u + 2v, 2u - v \rangle$

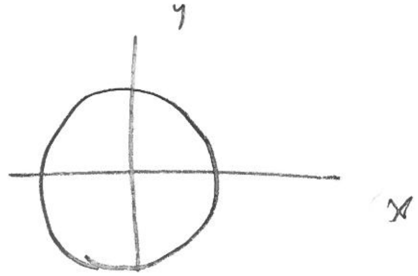
$\bar{r}_u = \langle -3, 2 \rangle = \langle -3, 2, 0 \rangle$

$\times \quad \bar{r}_v = \langle 2, -1 \rangle = \langle 2, -1, 0 \rangle$

$\langle 0, 0, -1 \rangle$

$$\boxed{\left| \frac{\partial(x, y)}{\partial(u, v)} \right| = 1 \,!}$$

$$\iint_{S} dS$$



$$dS = \|\bar r_x \times \bar r_y\| \, dx\,dy$$

$$\bar r = \langle x, y, 5-x^2-y^2 \rangle$$

$$\bar r_x = \langle 1, 0, -2x \rangle$$

$$\times \; \bar r_y = \langle 0, 1, -2y \rangle$$

$$\bar r_x \times \bar r_y = \langle 2x, 2y, 1 \rangle$$

$$\bar r = \langle r\cos\theta, r\sin\theta, 5-r^2 \rangle$$

$$\bar r_\theta = \langle -r\sin\theta, r\cos\theta, 0 \rangle$$

$$\bar r_r = \langle \cos\theta, \sin\theta, -2r \rangle$$

$$\times$$

$$\bar r_\theta \times \bar r_r = \langle -2r^2\cos\theta, -2r^2\sin\theta, -r\sin^2\theta - r\cos^2\theta \rangle$$

$$= \langle -2r^2\cos\theta, -2r^2\sin\theta, -r^2 \rangle$$

$$= \langle -2r^2\cos\theta, -2r^2\sin\theta, -r^2 \rangle$$

$$\Rightarrow \|\bar r_\theta \times \bar r_r\| = \sqrt{4r^4\cos^2\theta + 4r^4\sin^2\theta + r^4}$$

$$= r^2\sqrt{4r^2+1}$$

$$\bar{r}_x \times \bar{r}_y = \left\langle \frac{x}{\sqrt{1-x^2-y^2}}, \frac{y}{\sqrt{1-x^2-y^2}}, 1 \right\rangle$$

$$= \left\langle \frac{r\cos\theta}{\sqrt{1-r^2}}, \frac{r\sin\theta}{\sqrt{1-r^2}}, 1 \right\rangle$$

$$F \cdot (\bar{r}_x \times \bar{r}_y)$$

$$\frac{2r^3\cos^2\theta\sin\theta e^{\sqrt{1-r^2}} - r^3(1-\cos^2\theta)\sin\theta e^{\sqrt{1-r^2}}}{LCD}$$

$$= \frac{2r^3\cos^2\theta\sin\theta e^{\sqrt{1-r^2}} - r^3\sin\theta e^{\sqrt{1-r^2}} + r^3\cos^2\theta\sin\theta e^{\sqrt{1-r^2}}}{LCD}$$

$$= \frac{3r^3\cos^2\theta\sin\theta e^{\sqrt{1-r^2}} - r^3\sin\theta e^{\sqrt{1-r^2}}}{LCD}$$

$$= \frac{3r^3 e^{\sqrt{1-r^2}}\left(\cos^2\sin\theta - \sin\theta\right)}{\sqrt{1-r^2}}$$

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8a $\bar{F} = \langle 2xye^{z}, -y^2e^{z}, z \rangle$

$S_2: \quad \bar{r} = \langle x, y, \sqrt{1-x^2-y^2} \rangle$

$\bar{r}_x = \langle 1, 0, \frac{-x}{\sqrt{1-x^2-y^2}} \rangle$

$\times \quad \bar{r}_y = \langle 0, 1, \frac{-y}{\sqrt{1-x^2-y^2}} \rangle$

$\langle \frac{x}{\sqrt{1-x^2-y^2}}, \frac{y}{\sqrt{1-x^2-y^2}}, 1 \rangle$

$$\iint_{S_2} \bar{F} \cdot d\bar{S} = \iint_{S_2} \bar{F} \cdot \bar{n}\, dS = \iint \bar{F} \cdot \frac{\bar{r}_x \times \bar{r}_y}{\|\bar{r}_x \times \bar{r}_y\|} \left(\|\bar{r}_x \times \bar{r}_y\|\right) dA$$

$$= \iint \langle 2xye^{z}, -y^2e^{z}, z \rangle \cdot \langle \frac{x}{\sqrt{1-x^2-y^2}}, \frac{y}{\sqrt{1-x^2-y^2}}, 1 \rangle\, dy\,dx$$

$$= \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2x^2y\, e^{\sqrt{1-x^2-y^2}} - y^3 e^{\sqrt{1-x^2-y^2}}}{\sqrt{1-x^2-y^2}} + \sqrt{1-x^2-y^2}\, dy\,dx$$

$$= \int_0^{2\pi} \int_0^1 \left(\frac{2r^3 \cos^2\theta \sin\theta\, e^{\sqrt{1-r^2}} - r^3 \sin^3\theta\, e^{\sqrt{1-r^2}}}{\sqrt{1-r^2}} + \sqrt{1-r^2} \right) r\,dr\,d\theta$$