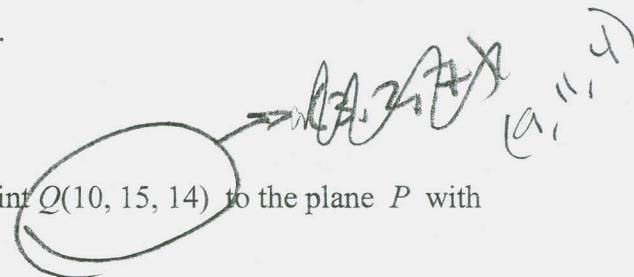


1. Suppose  $f(x, y) = 500 - x^2 - 3y^2$  describes a 500-foot dome in the Adirondacks.
  - a. (5 pts) What is the gradient, when you're standing at the point  $(10, 10, 100)$ ?
  - b. (5 pts) What is the directional derivative  $D_{\bar{u}}$  in the direction of  $\bar{u} = \langle 3, 4 \rangle$  at the point  $(10, 10, 100)$ ?

**Bonus** (5 pts) Use a vector to describe the direction in which you might start walking in order to neither gain nor lose altitude. Express this direction as a vector. There are actually 2 different directions you could take.

2. Distance from a point to a plane.
    - a. (5 pts) Find the distance from the point  $Q(10, 15, 14)$  to the plane  $P$  with equation  $2x + 3y - z = 5$
    - b. (10 pts) Find the point on the plane  $P$  that is closest to the point  $Q$  without using Lagrange multipliers.
    - c. (10 pts) Find the point on the plane  $P$  that is closest to the point  $Q$  by using Lagrange multipliers.
- 

## 203 TEST 2 Pt II

(1)

$$\textcircled{1} \quad f(x,y) = z = 500 - x^2 - 3y^2$$

$$(a) \nabla f = \langle -2x, -6y \rangle$$

$$\nabla f(10, 10) = \langle -20, -60 \rangle$$

$$(b) \bar{u} = \langle 3, 4 \rangle \rightarrow$$

$$|\bar{u}| = \sqrt{3^2 + 4^2} = 5$$

$$\nabla f(10, 10) \cdot \frac{\bar{u}}{|\bar{u}|} = \frac{1}{5} \langle -20, -60 \rangle \cdot \langle 3, 4 \rangle$$

$$= \frac{1}{5} (-60 - 240) = \frac{1}{5} (-300) = \boxed{-60}$$

Bonus Tangent to the contour @  $(10, 10, 100)$  is orthogonal to the gradient @  $(10, 10, 100)$ , so one method is solving

$$\nabla f(10, 10) \cdot \bar{u} = 0 \rightarrow$$

$$\langle -20, -60 \rangle \cdot \langle x, y \rangle = 0 \rightarrow$$

$$-20x - 60y = 0 \rightarrow$$

$$-20x = 60y \Rightarrow$$

$$x = -3y, \text{ so } \bar{u} = \langle -3y, y \rangle$$

$\Rightarrow \langle -3, 1 \rangle$  would work. So would

$\bar{u} = \langle 3, -1 \rangle$  and many others

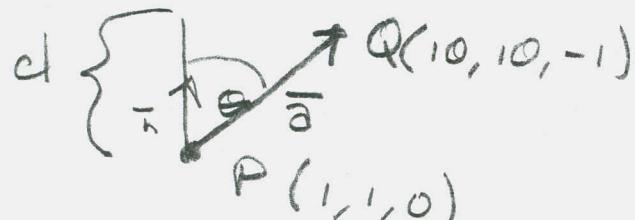
203 TEST 2 Pt II

(22) Distance from  $Q(10, 10, -1)$  to the plane (2)

$$2x + 3y - z = 5$$

Pick a point on the plane  $A(1, 1, 0)$  or  $P(0, 0, -5)$  are easy choices.

Use  $P(1, 1, 0)$ :



$$\vec{n} = \langle 2, 3, -1 \rangle$$

$$\bar{a} = \langle 9, 9, -1 \rangle$$

$$d = \frac{\bar{a} \cdot \vec{n}}{|\bar{a}|} = \frac{|\bar{a} \cdot \vec{n}|}{|\bar{a}| |\vec{n}|} \Rightarrow d = \frac{|\bar{a} \cdot \vec{n}|}{|\vec{n}|}$$

$$= \frac{|\langle 9, 9, -1 \rangle \cdot \langle 2, 3, -1 \rangle|}{\sqrt{9^2 + 9^2 + 1^2}} = \frac{|18 + 27 + 1|}{\sqrt{14}}$$

$$= \frac{46}{\sqrt{14}} = \frac{46}{\sqrt{14}}$$

203 Test 2 Pt II

(2b) Minimize  $(x-9)^2 + (y-11)^2 + (z-4)^2$  (3)  
 S.t.  $z = 2x+3y-5$

$$f(x,y) = (x-9)^2 + (y-11)^2 + (2x+3y-9)^2$$

$$\begin{aligned} \Rightarrow f_x &= 2(x-9) + 2(2x+3y-9)(2) \\ &= 2x-18 + 4(2x+3y-9) \\ &= 2x-18 + 8x + 12y - 36 \\ &= 10x + 12y - 54 \stackrel{\text{SET}}{=} 0 \end{aligned}$$

$$\Rightarrow \boxed{5x+6y = 27 \text{ E1}}$$

$$f_y = 2(y-11) + 2(2x+3y-9)(3)$$

$$= 2y-22 + 6(2x+3y-9)$$

$$= 2y-22 + 12x + 18y - 54$$

$$= 12x + 20y - 76 \stackrel{\text{SET}}{=} 0$$

$$\Rightarrow \boxed{3x+5y = 19 \text{ E2}}$$

$(x, y, z)$

$= (3, 2, 7)$

minimizes  
the distance.

SOLVE E1 system:

E2

$$(5x+6y = 27)(-3) \rightarrow -15x - 18y = -81$$

$$(3x+5y = 19)(5) \rightarrow \underline{15x + 25y = 95}$$

$$-3E1 + 5E2$$

$$7y = 14$$

$$y = 2 \Rightarrow 3x+5(2) = 19$$

$$\Rightarrow 3x = 9 \rightarrow x = 3$$

$$y = 2$$

$$\Rightarrow z = 6+6-5 = 7 = z$$

203 Test 2 pt II

(20)

$$\begin{aligned} \text{Minimize } & (x-9)^2 + (y-11)^2 + (z-4)^2 = F(x, y, z) \\ \text{s.t. } & 2x + 3y - z = 5 \end{aligned}$$

Let  $G(x, y, z) = 2x + 3y - z$ . Then

$$\nabla F = \lambda \nabla G \Rightarrow$$

$$\langle 2(x-9), 2(y-11), 2(z-4) \rangle = \lambda \langle 2, 3, -1 \rangle$$

$$\Rightarrow \langle 2x-18, 2y-22, 2z-8 \rangle = \langle 2\lambda, 3\lambda, -\lambda \rangle$$

$$\Rightarrow x-9 = \lambda \quad 2y-22 = 3\lambda \quad 2z-8 = -\lambda$$

$$x = \lambda + 9$$

$$2y = 3\lambda + 22$$

$$2z = -\lambda + 8$$

$$y = \frac{3\lambda + 22}{2}$$

$$z = \frac{8-\lambda}{2}$$

$$\Rightarrow 2x + 3y - z = 5 \text{ is now}$$

$$2(\lambda + 9) + 3\left(\frac{3\lambda + 22}{2}\right) - \frac{8-\lambda}{2} = 5 \Rightarrow$$

$$2\lambda + 18 + \frac{9\lambda + 66}{2} + \frac{\lambda - 8}{2} = 5 \Rightarrow$$

$$4\lambda + 36 + 9\lambda + 66 + \lambda - 8 = 10 \Rightarrow$$

$$14\lambda + 94 = 10$$

$$14\lambda = -84$$

$$\lambda = \frac{-84}{14} = -\frac{42}{7} = -6 = \lambda, \text{ so,}$$

$$x = -6 + 9 = 3 = x, y = \frac{3(-6) + 22}{2} = 2 = y, z = \frac{8+6}{2} = 7 = z$$

$$\boxed{(x, y, z) = (3, 2, 7)}$$