MAT 203 | Test 2 Pt. I
15.1 – 15.6, 65 pts | Spring, 2012 | Name KEY

1. (10 pts) Find and sketch the domain of $f(x, y) = \dfrac{\sqrt{x - y}}{\ln x}$.

2. The following problems involve limits of a function of two variables. The more convincing you are, the more points you will earn.

   a. (5 pts) Use the fact that $\sqrt{x^2 + y^2} \ge \sqrt{x^2}$ to show that

   $$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + y^2}} = 0.$$

   b. (5 pts) Evaluate $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^3 - x^2y + 2xy^2 - y^3}{x^2 + y^2}$.

3. Find the first partials f_x and f_y for $f(x, y)$.

   a. (5 pts) $f(x, y) = \arcsin\left(y\sqrt{x}\right)$

   b. (5 pts) $f(x, y) = \displaystyle\int_x^y \sqrt{1 - w^2}\, dw$

4. (10 pts) Find $\dfrac{\partial z}{\partial x}$, if $x^2 - y^2 - z = 3xz$.

5. Let $z = x^2 + 2y^2$

   a. (5 pts) Find an equation of the tangent plane to $f(x, y) = x^2 + 2y^2$ at $(1,1,3)$.

   b. (5 pts) Let $F(x, y, z) = x^2 + 2y^2 - z$, and find an equation of the tangent plane to the level surface $F(x, y, z) = 0$ at the point $(1,1,3)$.

   c. (5 pts) Use your answer to the previous problem to approximate $f(1.1,1.1)$.

   d. (5 pts) Find the *exact* value of $f(1.1,1.1)$, and use it to find the corresponding change in f from $(x, y) = (1,1)$ to $(x, y) = (1.1,1.1)$. That is, find Δz.

   e. (5 pts) Find the differential dz, and use it to approximate Δz from the previous problem.

Bonus (5 pts) What property of f explains the difference between dz and Δz?

Bonus (5 pts) Let P be the plane you found in #5a (or #5b). Find parametric equations for a normal line to the plane P that passes through the point $\left(2, \pi, \sqrt{2\pi}\right)$.

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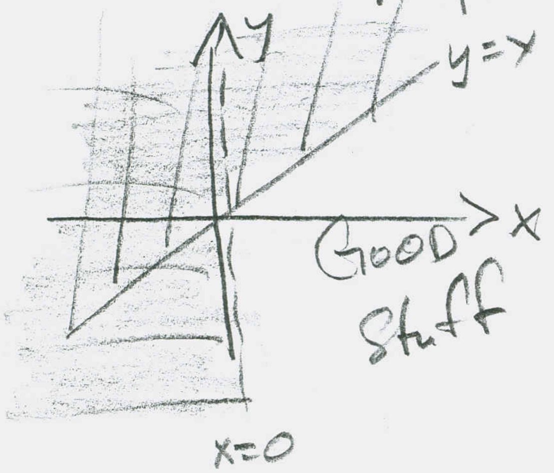
(1) $f(x,y) = \dfrac{\sqrt{x-y}}{\ln x} \longrightarrow$

$\mathcal{D} = \{ (x,y) \mid x-y \geq 0 \ \& \ x > 0 \}$



$x - y \geq 0$

$-y \geq -x$

$y \leq x$

$\cancel{\&}$ $x > 0$

(2a) $\left| \dfrac{xy}{\sqrt{x^2+y^2}} - 0 \right| = \left| \dfrac{xy}{\sqrt{x^2+y^2}} \right| \leq \left| \dfrac{xy}{\sqrt{x^2}} \right|$

$= \dfrac{xy}{|x|} = \begin{cases} y & \text{if } x > 0 \\ -y & \text{if } x < 0 \end{cases} \xrightarrow{(x,y)\to(0,0)} 0 \ \blacksquare$

(2b) $\dfrac{2x^3 - x^2y + 2xy^2 - y^3}{x^2+y^2} = \dfrac{x^2(2x-y) + y^2(2x-y)}{x^2+y^2}$

$= \dfrac{(2x-y)(x^2+y^2)}{x^2+y^2} = 2x - y \xrightarrow{(x,y)\to(0,0)} 0 \ \blacksquare$

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(3) $f(x,y) = \arcsin(y\sqrt{x}) \longrightarrow$

(a) $f_x = \dfrac{1}{\sqrt{1-(y\sqrt{x})^2}} \cdot \dfrac{y}{2\sqrt{x}}$

$f_y = \dfrac{1}{\sqrt{1-(y\sqrt{x})^2}} \cdot \sqrt{x}$

(b) $f(x,y) = \int_x^y \sqrt{1-w^2}\ dw \longrightarrow$

$f_x = -\sqrt{1-w^2}$

$f_y = \sqrt{1-w^2}$

(4) $x^2 - y^2 z = 3xz$. Find $\frac{dz}{dx}$.

Let $F(x,y,z) = x^2 - y^2 - z - 3xz$. Then

$$\frac{dz}{dx} = -\frac{F_x}{F_z} = \boxed{-\frac{2x-3z}{-1-3x} = \frac{dz}{dx}}$$

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5 $z = x^2 + 2y^2$

(a) tan plane @ $(1,1,3)$ is

$z = f_x(x-1) + f_y(y-1) + 3$

$f_x = 2x \implies f_x(1,1) = 2$

$f_y = 4y \implies f_y(1,1) = 4$

So $\boxed{z = 2(x-1) + 4(y-1) + 3}$

(b) $F(x,y,z) = x^2 + 2y^2 - z$

$F_x = 2x \implies F_x(1,1,3) = 2$

$F_y = 4y \implies F_y(1,1,3) = 4$

$F_z = -1 \implies F_z(1,1,3) = -1$

So $F_x(x-1) + F_y(x-1) + F_z(z-3) = 0$

$\implies \boxed{2(x-1) + 4(y-1) - 1(z-3) = 0}$

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5c $f(1.1, 1.1) \approx 2(1.1-1) + 4(1.1-1) + 3$

$= 2(.1) + 4(.1) + 3$

$= .2 + .4 + 3$

$= .6 + 3$

$= 3.6 \approx f(1.1, 1.1)$

5d $f(1.1, 1.1) = (1.1)^2 + 2(1.1)^2$

$= 1.21 + 2(1.21) = 3(1.21) = \boxed{3.63}$

$\therefore \Delta z = f(1.1, 1.1) - f(1, 1)$

$= 3.63 - [1^2 + 3(1)^2]$

$= 3.63 - 3$

$= \boxed{.63 = \Delta z}$

5e $dz = 2x\,dx + 4y\,dy \Longrightarrow$ @

$= 2(1)(.1) + 4(1)(.1)$

$= .2 + .4 = \boxed{.6 = dz}$

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Bonus 1

$\Delta z = .63 > dz = .6,$

because $\boxed{f(x,y) \text{ is concave up.}}$

Linearization below the surface.

Bonus 2

$2(x-1) + 4(y-1) - (z-3) = 0$

$\vec{n} = \langle 2, 4, -1 \rangle$ so a line in that direction containing $(2, \pi, \sqrt{2\pi})$ is given by

$$\boxed{\begin{aligned} x &= 2 + 2t \\ y &= \pi + 4t \\ z &= \sqrt{2\pi} - t \end{aligned}}$$