MAT 203 Test 1
Chapters 13, 14 Spring, 2012 Name______________________

Do all your work on separate paper. You do not have to re-state the question, as we expect in the homework, but submit all the problems, in order, and clearly numbered, so your poor teacher can award all the credit you so richly deserve.

1. Find parametric equations of the line L where the planes $\begin{matrix} 2x - 6y + 8z = -16 \\ 2x - 5y + 7z = -14 \end{matrix}$ meet.

2. Let $A(1,-1,3), B(2,3,1), C(1,0,4)$, and $D(0,3,2)$.

   a. Find the vectors $\bar{u} = \overrightarrow{AB}, \bar{v} = \overrightarrow{AC}$, and $\bar{w} = \overrightarrow{AD}$.

   b. Find parametric equations of the line containing the two points, A and B.

   c. Find a *vector* function $\bar{r}(t)$ describing the directed line segment from A to B.

   Construct it so that $\bar{r}(0) = \bar{r}_0 = \langle 1,-1,3\rangle$ and $\bar{r}(1) = \bar{r}_1 = \langle 2,3,1\rangle$.

   d. Find the area of the parallelogram defined by the vectors $\bar{u}$ and $\bar{v}$.

   e. Find the volume of the parallelepiped defined by the vectors $\overrightarrow{AB}, \overrightarrow{AC}$, and $\overrightarrow{AD}$

   f. Find an equation of the plane P containing the 3 points A, B, and C in the form $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

3. Distance Problems:

   a. Find the distance from the plane P to the point D.

   b. Find the distance from the line L to the point D.

4. The path of a particle moving through space is described by the vector function $\bar{r}(t) = \langle 5\sin(t), 3\cos(t), 4\cos(t)\rangle$, for $0 \le t \le 2\pi$.

   a. Find $\bar{r}'(t)$ and $|r'(t)| = s'(t)$.

   b. Find the arc length function, $s(t)$, describing the distance traveled by the particle, for $0 \le t \le 2\pi$. What's the distance traveled from $t = 0$ to $\frac{\pi}{2}$?

   c. Find the unit tangent, $\bar{T}(t)$, unit normal, $\bar{N}(t)$, and the unit binormal, $\bar{B}(t)$.

If you have time, try one of these on for size:

Bonus 1. Sketch the projections of $\bar{r}(t)$ into the *xy*- *xz*- and *yz*-planes, using 3 separate sketches. Give a verbal description of the graph of $\bar{r}(t)$, and try to render a sketch of it in 3 dimensions. Hint: In the *xy*- plane, you're sketching the curve with parametric equations $x(t) = 5\sin(t),\ y(t) = 3\cos(t)$.

Bonus 2. Find the radius of the osculating circle to $\bar{r}(t)$ at $\bar{r}\left(\frac{\pi}{6}\right) = \left\langle \frac{5}{2}, \frac{3\sqrt{3}}{2}, \frac{4\sqrt{3}}{2} \right\rangle$