Name: ______________________ Class: __________________ Date: _________ ID: A

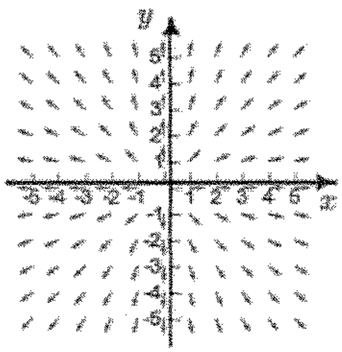
Final Test (Chapter 16

Short Answer

1. Evaluate the line integral over the given curve C.

   $\int_C 4xy\,ds$, where C is the line segment joining (–4, –5) to (5, 4)

2. The plot of a vector field is shown below. A particle is moved from the point $(3,3)$ to $(0,0)$. By inspection, determine whether the work done by $\mathbf{F}$ on the particle is positive, negative, or zero.



3. Evaluate the line integral over the given curve C.

   $\int_C 4xy\,ds$, where C is the line segment joining (–2, –1) to (4, 5)

4. Determine whether $\mathbf{F}$ is conservative. If so, find a function f such that $\mathbf{F} = \nabla f$..

   $\mathbf{F}(x,y,z) = 9x^2y^4z^2\mathbf{i} + 12x^3y^3z^2\mathbf{j} + 6x^3y^4z\mathbf{k}$

5. Determine whether $\mathbf{F}$ is conservative. If so, find a function f such that $\mathbf{F} = \nabla f$..

   $\mathbf{F}(x,y,z) = (6\sinh 2z)\mathbf{i} + \left(3e^{5z}\cos 3y\right)\mathbf{j} + (12x\cosh 2z)\mathbf{k}$

Name: ______________________ ID: A

6. Let R be a plane region of area A bounded by a piecewise-smooth simple closed curve C. Using Green's Theorem, it can be shown that the centroid of R is $\left(\bar{x},\bar{y}\right)$, where

$$\bar{x} = \frac{1}{2A}\oint_C x^2 dy \qquad\qquad \bar{y} = -\frac{1}{2A}\oint_C y^2 dx$$

Use these results to find the centroid of the given region.

The triangle with vertices $\left(0,0\right)$, $\left(3,0\right)$, and $\left(3,4\right)$.

7. Find (a) the divergence and (b) the curl of the vector field **F**.

$\mathbf{F}\left(x,y,z\right) = \cos z\mathbf{i} + 5y\sin 3z\mathbf{j} + 4x^2z\mathbf{k}$

8. Let f be a scalar field. Determine whether the expression is meaningful. If so, state whether the expression represents a scalar field or a vector field.

curl f

9. Let **F** be a vector field. Determine whether the expression is meaningful. If so, state whether the expression represents a scalar field or a vector field.

$\nabla \cdot (\nabla \times \mathbf{F})$

10. Find the area of the surface S where S is the part of the plane $z = 2x^2 + y$ that lies above the triangular region with vertices (0, 0), (3, 0), and (3, 3).

11. Find the area of the surface S where S is the part of the sphere $x^2 + y^2 + z^2 = 16$ that lies to the right of the xz-plane and inside the cylinder $x^2 + z^2 = 9$.

12. Find the area of the surface S where S is the part of the sphere $x^2 + y^2 + z^2 = 1$ that lies inside the cylinder $x^2 - x + y^2 = 0$.

13. Find an equation in rectangular coordinates, and then identify the surface.

$r\left(u,v\right) = 6v\mathbf{i} + (8u - v)\mathbf{j} + (u + 6v)\mathbf{k}$

14. Find a vector representation for the surface.

The plane that passes through the point $(2,5,1)$ and contains the vectors $2\mathbf{i}+5\mathbf{j}-3\mathbf{k}$ and $2\mathbf{i}-3\mathbf{j}+5\mathbf{k}$..

15. Find an equation of the tangent plane to the parametric surface represented by $\mathbf{r}$ at the specified point.

$\mathbf{r}(u,v)=\left(u^2-v^2\right)\mathbf{i}+u\mathbf{j}+v\mathbf{k};\ (0,3,3)$

16. Find an equation of the tangent plane to the parametric surface represented by $\mathbf{r}$ at the specified point.

$\mathbf{r}(u,v)=ue^{v}\mathbf{i}+uv\mathbf{j}+ve^{-u}\mathbf{k};\ u=\ln 5,\ v=0$

17. Find an equation of the tangent plane to the parametric surface represented by $\mathbf{r}$ at the specified point.

$\mathbf{r}(u,v)=ue^{v}\mathbf{i}+uv\mathbf{j}+ve^{-u}\mathbf{k};\ u=\ln 9,\ v=0$

18. Use the Divergence Theorem to find the flux of $\mathbf{F}$ across S; that is, calculate $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$.

$\mathbf{F}(x,y,z)=(9xy+\cos z)\mathbf{i}+(x-\sin z)\mathbf{j}+4xz\mathbf{k}$; S is the sphere $x^2+y^2+z^2=4$

19. Use Stokes' Theorem to evaluate $\iint_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S}$.

$\mathbf{F}(x,y,z)=4xy\mathbf{i}+5yz\mathbf{j}+2z^2\mathbf{k};$

S is the part of the ellipsoid $9x^2+9y^2+4z^2=36$ lying above the xy-plane and oriented with normal pointing upward.

20. Use Stokes' Theorem to evaluate $\oint_C \mathbf{F}\cdot d\mathbf{r}$.

$\mathbf{F}(x,y,z)=7z\mathbf{i}+y\mathbf{j}+4xz\mathbf{k};$

C is the boundary of the triangle with vertices $(6,0,0)$, $(0,6,0)$, and $(0,0,6)$ oriented in a counterclockwise direction when viewed from above

Final Test (Chapter 16
Answer Section

SHORT ANSWER

1. ANS:
$234\sqrt{2}$

PTS: 1 DIF: Medium REF: 16.2.4 MSC: Short Answer
NOT: Section 16.2

2. ANS:
negative

PTS: 1 DIF: Medium REF: 16.2.17a MSC: Short Answer
NOT: Section 16.2

3. ANS:
$120\sqrt{2}$

PTS: 1 DIF: Medium REF: 16.2.2 MSC: Short Answer
NOT: Section 16.2

4. ANS:
$f\left(x,y,z\right) = 3x^3y^4z^2 + C$

PTS: 1 DIF: Easy REF: 16.3.3 MSC: Short Answer
NOT: Section 16.3

5. ANS:
The vector field $\mathbf{F}\left(x,y,z\right) = (6\sinh 2z)\mathbf{i} + \left(3e^{5z}\cos 3y\right)\mathbf{j} + (12x\cosh 2z)\mathbf{k}$ is not conservative. There exists no scalar field f such that $\mathbf{F} = \nabla f$.

PTS: 1 DIF: Easy REF: 16.3.10 MSC: Short Answer
NOT: Section 16.3

6. ANS:
$\bar{x} = 2; \bar{y} = \dfrac{4}{3}$

PTS: 1 DIF: Medium REF: 16.4.23 MSC: Short Answer
NOT: Section 16.4

7. ANS:
(a). $4x^2 + 5\sin 3z$
(b). $-15y\cos 3z\mathbf{i} - \left(8xz + \sin z\right)\mathbf{j}$

PTS: 1 DIF: Medium REF: 16.5.4 MSC: Short Answer
NOT: Section 16.5

8. ANS:
The curl is a property of vector fields, not scalar fields. So, curl f is not meaningful.

PTS: 1 DIF: Medium REF: 16.5.12a MSC: Short Answer
NOT: Section 16.5

9. ANS:
$\nabla \times \mathbf{F}$ is the curl of $\mathbf{F}$, so it is a vector field. Thus, $\nabla \cdot (\nabla \times \mathbf{F})$ is the divergence of a vector field, which is a scalar field. Assuming all the partial derivatives are defined and continuous, $\nabla \cdot (\nabla \times \mathbf{F})$ is meaningful.

PTS: 1 DIF: Medium REF: 16.5.121 MSC: Short Answer
NOT: Section 16.5

10. ANS:
$$\frac{73\sqrt{146}-\sqrt{2}}{24}$$

PTS: 1 DIF: Medium REF: 16.6.44 MSC: Short Answer
NOT: Section 16.6

11. ANS:
$16\pi\left(8-\sqrt{55}\right)$

PTS: 1 DIF: Difficult REF: 16.6.45 MSC: Short Answer
NOT: Section 16.6

12. ANS:
$2(\pi-2)$

PTS: 1 DIF: Difficult REF: 16.6.50 MSC: Short Answer
NOT: Section 16.6

13. ANS:
Answers may vary.
$49x+6y-48z=0$; plane

PTS: 1 DIF: Easy REF: 16.6.3 MSC: Short Answer
NOT: Section 16.6

14. ANS:
Answers may vary.
$\mathbf{r}\left(u,v\right)=(2+2u+2v)\mathbf{i}+(5+5u-3v)\mathbf{j}+(1-3u+5v)\mathbf{k}$

PTS: 1 DIF: Medium REF: 16.6.19 MSC: Short Answer
NOT: Section 16.6

15. ANS:
Answers may vary.
$x - 6y + 6z = 0$

PTS: 1 DIF: Medium REF: 16.6.38 MSC: Short Answer
NOT: Section 16.6

16. ANS:
Answers may vary.
$y - 5\ln 5z = 0$

PTS: 1 DIF: Medium REF: 16.6.35 MSC: Short Answer
NOT: Section 16.6

17. ANS:
Answers may vary.
$y - 9\ln 9z = 0$

PTS: 1 DIF: Medium REF: 16.6.36 MSC: Short Answer
NOT: Section 16.6

18. ANS:
0

PTS: 1 DIF: Difficult REF: 16.6.9 MSC: Short Answer
NOT: Section 16.9

19. ANS:
0

PTS: 1 DIF: Medium REF: 16.8.3 MSC: Short Answer
NOT: Section 16.8

20. ANS:
-18

PTS: 1 DIF: Medium REF: 16.8.4 MSC: Short Answer
NOT: Section 16.8