MAT 203 | Test 2 – Fall, 2018 | Name_______________________

70 Points | Covers Chapter 14 | NO GRAPHING CALCULATORS!!!

Do all your work and submit answers with your work, on the separate paper provided. Organize your work for efficient grading and feedback. Leave a margin, especially in the top left, where the staple goes!

1. (10 pts) Find and graph the domain of $f(x,y)=\sqrt{y-1}+\sqrt{25-x^2}$.

2. Find the first partials f_x and f_y for...

   a. (10 pts) $f(x,y)=\left(x^2-3xy+5y^{-4}\right)^2$

   b. (10 pts) $f(x,y)=\int_0^{x^2-5x}\left(\frac{\sin(\tau)\cosh(\tau)}{\tau^2+\pi}\right)d\tau$

3. Find $\frac{\partial z}{\partial x}$ for the equation $y\sin\left(xy^3\right)+x^2yz^2=2xyz$ in 2 ways:

   a. (5 pts) Use implicit differentiation, holding *y* constant and treating *z* as an implicit function of *x*.

   b. (5 pts) Form a function $F(x,y,z)$ and find $\frac{\partial z}{\partial x}$ for the level surface $F(x,y,z)=0$.

4. Let $f(x,y)=2x^2+4y^2+10$.

   a. (10 pts) Find an equation of the tangent plane to *f* at the point $(1,-1,f(1,-1))=(1,-1,16)$.

   b. (10 pts) Use your previous answer to approximate $f(1.2,-0.9)$.

   c. (5 pts) Find the actual value of $f(1.2,-0.9)$.

   d. (5 pts) Find Δz for the change in *z* from $f(1,-1)=16$ to $f(1.2,-0.9)$

   e. (5 pts) Find the differential approximation $dz\approx\Delta z$. You may calculate this, directly, or just use previous work and a subtraction.

   f. (5 pts) What is the gradient of *f* at $(1,-1,16)$?

   g. (5 pts) Find the directional derivative for *f*, $D_{\bar{u}}$ in the direction of $\bar{u}=\langle -3,2\rangle$ at the point $(1,-1,16)$

5. Find the shortest distance between the plane $2x - y + 3z = 6$ and the point $P(2,3,7)$ in three ways:

   a. (5 pts) Use 1st- and/or 2nd- derivative test.

   b. (5 pts) Use earlier skills from Chapter 12.

   c. (5 pts) Use Lagrange Multipliers.

Bonus: Answer up to 3 of the following for up to 15 bonus points.

1. (5 pts) (Line segment) Write the equation of the line segment between $A(1,2,3)$ and $B(-3,2,1)$.

2. (5 pts) Consider the object $9x^2 + 4z^2 - 25y = 0$. Show its traces in the planes $x = k, y = k, z = k$ for different choices of k and project those into the $yz-, xz-,$ and $xy-$ planes, respectively.

3. (5 pts) Give a verbal description of the statement $\kappa = \left|\frac{d\overline{T}}{ds}\right|$. What is it? What does it mean? What's our shortcut for calculating it, in terms of $\overline{r}(t)$?