203 TEST 2 SPRING 2019

① (10 pts) $f(x,y) = \sqrt{y-1} + \sqrt{25-x^2}$

$\mathcal{D}$: Need $y-1 \ge 0$ $\quad$ Need $25-x^2 \ge 0$

$\longrightarrow y \ge 1$ $\quad$ $\longrightarrow x^2-25 \le 0$

$\longrightarrow (x-5)(x+5) \le 0$

$+ \quad - \quad +$ $\quad$ ≤ 0

$-5 \quad 5$

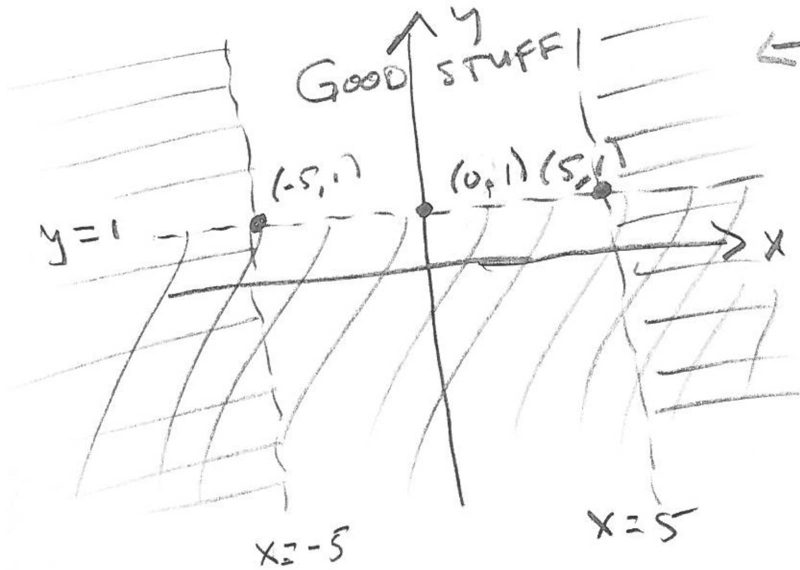
$=0 \quad =0$

N Y Y Y N

$-5 \le x \le 5$

$x \ge -5$

$x \le 5$



② ⓐ (10 pts) $f(x,y) = (x^2-3xy+5y^{-4})^2$

$\longrightarrow$ $f_x = 2(x^2-3xy+5y^{-4})(2x-3y)$ $\quad$ 10 pts

& $f_y = 2(x^2-3xy+5y^{-4})(-3x-20y^{-5})$

ⓑ $f(x,y) = \int_0^{x^2-5x} \left(\frac{\sin(\tau)\cosh(\tau)}{\tau^2+\pi}\right) d\tau$

$\longrightarrow$ $f_x = \left(\frac{\sin(x^2-5x)\cos(x^2-5x)}{x^2-5x+\pi}\right)(2x-5)$

& $f_y = 0$!

T2

$y \sin(xy^3) + x^2 y z^2 = 2xyz$

(a) 5pts

$$y\cos(xy^3)\cdot y^3 + 2xyz^2 + x^2y\cdot 2zz'$$
$$= 2yz + 2xyz'$$

$$\Rightarrow 2x^2yzz' - 2xyz'$$
$$= 2yz - y^4\cos(xy^3) - 2xyz^2$$

$$\Rightarrow z' = \boxed{\frac{dz}{dx} = \frac{2yz - y^4\cos(xy^3) - 2xyz^2}{2x^2yz - 2xy}}$$

(b) 5pts $F(x,y,z) = y\sin(xy^3) + x^2yz^2 - 2xyz$ set 0

$$\Rightarrow F_x = y^4\cos(xy^3) + 2xyz^2 - 2yz,$$
$$F_z = 2x^2yz - 2xy$$

$$\Rightarrow \frac{dz}{dx} = -\frac{F_x}{F_z} = \boxed{-\frac{y^4\cos(xy^3) + 2xyz^2 - 2yz}{2x^2yz - 2xy} = \frac{dz}{dx}}$$

T2

$f(x,y) = 2x^2 + 4y^2 + 10 = z$, $(1,-1,16) = (x_0, y_0, z_0) = P$

(a) $f_x = 4x \Longrightarrow f_x(1,-1) = 4 = f_x$

$f_y = 8y \longrightarrow f_y(1,-1) = -8 = f_y$

$\Rightarrow L(x,y) = f_x(x-x_0) + f_y(y-y_0) + z_0$

10pts $= \boxed{4(x-1) - 8(y+1) + 16 = z}$

(b) 10pts $f(1.2, -0.9)$

$\approx L(1.2,-0.9) = 4(1.2-1) - 8(-0.9+1) + 16$

$= 4(.2) - 8(.1) + 16 = .8 - .8 + 16 = \boxed{16 = L(1.2, -.9)}$

(c) 5pts $f(1.2, -.9) = 2(1.2)^2 + 4(-.9)^2 + 16$

$= 2(1.44) + 4(.81) + 16$

$= 2.88 + 3.24 + 16 = 6.12 + 16 = \boxed{22.12 = f(1.2, -.9)}$

(d) 5pts $\Delta z = f(x+\Delta x, y+\Delta y) - f(x,y)$

$= 22.12 - 16.00$

$= \boxed{6.12 = \Delta z}$

(e) 5pts $dz = L(x+\Delta x, y+\Delta x) - f(x,y) =$

$dz = f_x \Delta x + f_y \Delta y = 4(.2) - 8(.1) = .8 - .8$

$\boxed{= 0 = dz!}$

Not a good estimate.

203 T2

4f 5pts $\boxed{\nabla f = \langle f_x, f_y \rangle = \langle 4, -8 \rangle}$ @ $(x,y) = (1,-1)$

4g 5pts $D_{\bar{u}} = (\nabla f) \cdot \left(\frac{\bar{u}}{\|\bar{u}\|} \right)$

$\bar{u} = \langle -3, 2 \rangle \Rightarrow \|\bar{u}\| = \sqrt{9+4} = \sqrt{13}$

$\Rightarrow D_{\bar{u}} = \langle 4, -8 \rangle \cdot \frac{1}{\sqrt{13}} \langle -3, 2 \rangle$

$= \frac{1}{\sqrt{13}}(-12-16) = \boxed{\frac{-28}{\sqrt{13}} = D_{\bar{u}}}$

5 a 5pts $2x - y + 3z = 6 \Rightarrow y = 2x + 3z - 6$

$d((x,y,z),(2,3,7))$

$= \sqrt{(x-2)^2 + (y-3)^2 + (z-7)^2}$. Let $f(x,y) = d^2(x,y,z)$

$\Rightarrow f(x,y,z) = (x-2)^2 + (y-3)^2 + (z-7)^2$ to be minimized.

Define $g(x,z) = (x-2)^2 + (2x+3z-6-3)^2 + (z-7)^2$

$= (x-2)^2 + (2x+3z-9)^2 + (z-7)^2$

$\Rightarrow$

$g_x = 2(x-2) + 2(2x+3z-9)(2)$

$= 2x - 4 + 8x + 12z - 36 = 10x + 12z - 40 \overset{\text{SET}}{=} 0$

$\Rightarrow 5x + 6z - 20 = 0 \Rightarrow 5x = -6z + 20$

$\Rightarrow \boxed{x = \frac{-6z+20}{5}}$

203 T2

5a cnt'd

$$g_z = 2(2x+3z-9)(3) + 2(z-7)$$

$$= (4x+6z-18)(3) + 2z - 14$$

$$= 12x + 18z - 54 + 2z - 14 = 12x + 20z - 68 \overset{SET}{=} 0$$

$$\Rightarrow 6x + 10z - 34 = 0$$

$$\Rightarrow 6x = -10z + 34$$

$$\Rightarrow x = \frac{-10z+34}{6} = \boxed{\frac{-5z+17}{3} = x}$$

$$x = x \Rightarrow \frac{-5z+17}{3} = \frac{-6z+20}{5}$$

$$\Rightarrow -25z + 85 = -18z + 60$$

$$\Rightarrow -7z = -25$$

$$\Rightarrow \boxed{z = \frac{25}{7}}$$

$$\begin{array}{r} {}^{4}17 \\ 7 \\ \hline 119 \end{array}$$

$$\Rightarrow x = \frac{-5z+17}{3} = \frac{-5\left(\frac{25}{7}\right)+17}{3} = \frac{-\frac{125}{7} + \frac{119}{7}}{3}$$

$$= \frac{-6}{21} = \boxed{-\frac{2}{7} = x}$$

$$\Rightarrow y = 2x + 3z - 6 = 2\left(-\frac{2}{7}\right) + 3\left(\frac{25}{7}\right) - 6$$

$$= \frac{-4}{7} + \frac{75}{7} - \frac{42}{7} = \frac{75-46}{7} = \boxed{\frac{29}{7} = y}$$

203 T2

5a cnt'd

$$\Rightarrow d\left(-\tfrac{2}{7}, \tfrac{29}{7}, \tfrac{25}{7}\right) = \sqrt{\left(-\tfrac{2}{7}-2\right)^2 + \left(\tfrac{29}{7}-3\right)^2 + \left(\tfrac{25}{7}-7\right)^2}$$

$$= \sqrt{\left(\frac{-2-14}{7}\right)^2 + \left(\frac{29-21}{7}\right) + \left(\frac{25-49}{7}\right)^2}$$

$$= \frac{\sqrt{16^2 + 8^2 + 24^2}}{7} = \frac{\sqrt{256 + 64 + 576}}{7}$$

$$\begin{array}{r} 24 \\ \times\, 24 \\ \hline 96 \\ 480 \\ \hline 576 \end{array}$$

$$= \frac{\sqrt{896}}{7} = \boxed{\frac{8\sqrt{14}}{7} = d}$$

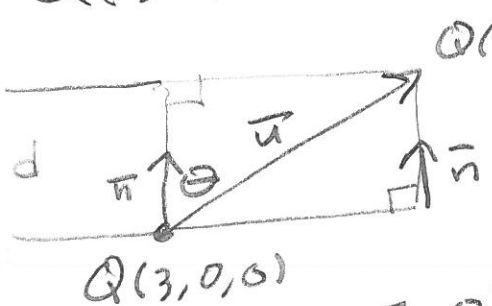
$$\begin{array}{r|l} 2 & 896 \\ 2 & 448 \\ 2 & 224 \\ 2 & 112 \\ 2 & 56 \\ 2 & 28 \\ 2 & 14 \\ & 7 \end{array}$$

$$\begin{array}{r} 256 \\ 64 \\ 576 \\ \hline 896 \end{array}$$

5b $2x - y + 3z = 6 : \mathcal{P}$

$P(2,3,7)$, $\bar{n} = \langle 2,-1,3 \rangle$

$Q(3,0,0) \in \mathcal{P}$



$$\frac{d}{\|\bar{u}\|} = \cos\theta = \frac{\bar{u}\cdot\bar{n}}{\|\bar{n}\|\,\|\bar{u}\|} \Rightarrow$$

$\bar{u} = \langle 2-3, 3-0, 7-0 \rangle = \langle -1, 3, 7 \rangle = \bar{u}$

$$\frac{d}{\|\bar{u}\|} = \cos\theta = \frac{\bar{u}\cdot\bar{n}}{\|\bar{u}\|\,\|\bar{n}\|} \Rightarrow$$

$$d = \frac{\bar{u}\cdot\bar{n}}{\|\bar{n}\|} = \frac{|\langle -1,3,7\rangle \cdot \langle 2,-1,3\rangle|}{\sqrt{2^2+1^2+3^2}} = \frac{|-2-3+21|}{\sqrt{14}}$$

Sweet!

$$= \frac{16}{\sqrt{14}} = \frac{16\sqrt{14}}{14} = \boxed{\frac{8\sqrt{14}}{7} = d}$$

203 T2

5c $f(x,y,z) = (x-2)^2 + (y-3)^2 + (z-7)^2$

$\Rightarrow \nabla f = \langle 2(x-2), 2(y-3), 2(z-7) \rangle$

$g(x,y,z) = 2x - y + 3z \longrightarrow$

$\nabla g = \langle 2, -1, 3 \rangle$

SET $\nabla f = \lambda g$:

$2x - 4 = 2\lambda \quad 2y - 6 = -\lambda \quad 2z - 14 = 3\lambda$

$2x = 2\lambda + 4 \quad 2y = -\lambda + 6 \quad 2z = 3\lambda + 14$

$\boxed{x = \lambda + 2} \quad \boxed{y = -\frac{1}{2}\lambda + 3} \quad \boxed{z = \frac{3}{2}\lambda + 7}$

$\longrightarrow 2x - y + 3z = 2(\lambda + 2) - (-\frac{1}{2}\lambda + 3) + 3(\frac{3}{2}\lambda + 7) \overset{SET}{=} 6$

$= 2\lambda + 4 + \frac{1}{2}\lambda - 3 + \frac{9}{2}\lambda + 21$

$\longrightarrow \frac{4+1+9}{2}\lambda = 6 - 4 + 3 - 21 = 9 - 25 = -16$

$\Rightarrow \frac{14\lambda}{2} = -16 \longrightarrow \lambda = -\frac{16}{7} = \boxed{-\frac{16}{7} = \lambda}$

$\longrightarrow x = \lambda + 2 = -\frac{16}{7} + \frac{14}{7} = \boxed{-\frac{2}{7} = x}$

$y = -\frac{1}{2}\lambda + 3 = -\frac{1}{2}(-\frac{16}{7}) + \frac{21}{7} = \frac{8}{7} + \frac{21}{7} = \boxed{\frac{29}{7} = y}$

$= \boxed{\frac{29}{7} = y}$

$z = \frac{3}{2}\lambda + 7 = \frac{3}{\cancel{2}}(-\frac{\overset{8}{\cancel{16}}}{7}) + \frac{49}{7} = \frac{-24 + 49}{7} = \boxed{\frac{25}{7} = z}$

$\boxed{(x,y,z) = (-\frac{2}{7}, \frac{29}{7}, \frac{25}{7})}$

203 T2

Bonus

1 5pts $A(1,2,3) \rightsquigarrow$ $\boxed{\bar{u} = \langle 1,2,3 \rangle}$

$B(-3,2,1) \rightsquigarrow$ $\boxed{\bar{v} = \langle -3,2,1 \rangle}$

$\longrightarrow$ Line segment from A to B is given in vector form by $\bar{r}(t)$, where

$$\boxed{\bar{r}(t) = (1-t)\bar{u} + t\bar{v}} \quad \boxed{\forall\, t \in [0,1]}$$

$$= (1-t)\langle 1,2,3 \rangle + t\langle -3,2,1 \rangle$$

$$= \langle 1,2,3 \rangle - \langle t, 2t, 3t \rangle + \langle -3t, 2t, t \rangle$$

$$= \langle 1-t-3t,\ 2-2t+2t,\ 3-3t+t \rangle$$

$$= \langle 1-4t, 2, 3-2t \rangle = \bar{r}(t)$$ if you can't help yourself & stop when you should.

203 T2

Bonus 2

5pts

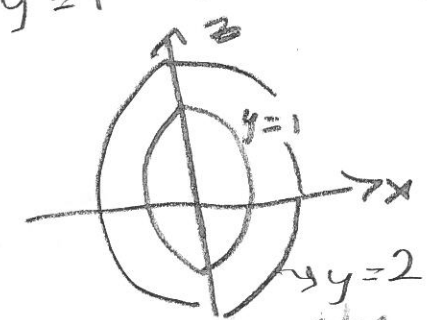
$$9x^2+4z^2-25y=0$$

<u>xz-plane</u>

$$25y = 9x^2+4z^2$$

$$y = \frac{x^2}{\left(\frac{25}{9}\right)} + \frac{z^2}{\frac{25}{4}} = \frac{x^2}{\left(\frac{5}{3}\right)^2} + \frac{z^2}{\left(\frac{5}{2}\right)^2}$$

$y=1$ is an ellipse that's tall & skinny:



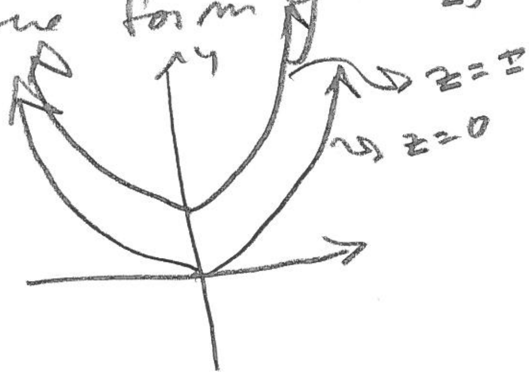
As y increases, the shape is same, but grows larger, coming out of the paper more and more.

@ $y=0$, $x=0=z$ is necessary as well from $a x^2 = -b z^2$

xy-plane:

$25y = 9x^2 + 4z^2$ OR

$y = \frac{9}{25}x^2 + \frac{4}{25}z^2$. All parabolas of the form $y = \frac{9}{25}x^2 + \frac{4}{25}K^2$ for all choices $z=K$.



203 T2

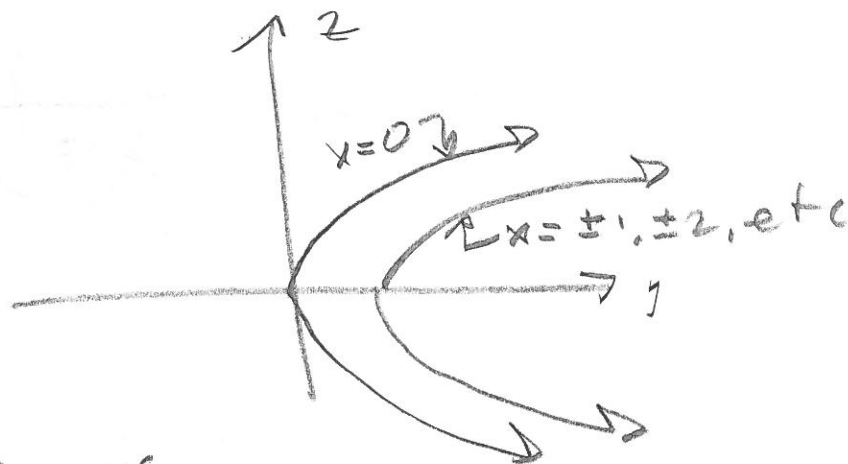
Bonus 2 cnt'd

yz-plane -

$25y = 9x^2 + 4z^2$ OR

$$y = \frac{4z^2}{25} + \frac{9x^2}{25}$$

Basically the same as the xy-plane, only broader/fatter parabolas, because $\frac{4}{25}z^2$ + STUFF is shorter than $\frac{9}{25}x^2$ + stuff



Bonus

3 5pts $\kappa = \left|\frac{d\bar{T}}{ds}\right|$ is the rate of change in the UNIT TANGENT with respect to arc length. Basically, how much do you turn as a function of how far forward you've gone. The tighter the corner the "bigger" the curvature.

$$\boxed{\kappa = \frac{\|\bar{r}' \times \bar{r}''\|}{\|\bar{r}'\|^3}}$$ is the computational formula