TEST 1 SPRING, 2019

(1) 10 pts

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -4 \\ 2 & 1 & -4 & -3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & -4 \\ 0 & -1 & -2 & 5 \end{array}\right]$$

$-y-2z=5$

$-y=2z+5$

$y=-2z-5$

$x+y-z=-4$

$x+(-2z-5)-z=-4$

$x-2z-5-z=-4$

$x-3z=1$

$x=3z+1$

$$\boxed{\begin{aligned} x &= 3t+1 \\ y &= -2t-5 \\ z &= t \end{aligned}}$$

Parametric Eq'ns

$\boxed{\bar{v} = \langle 3,-2,1\rangle}$ is direction vector, $\boxed{\bar{r}_0=\langle 1,-5,0\rangle}$ is a position vector (point) on the line, so $\boxed{\bar{r}(t) = \bar{r}_0 + t\bar{v}}$ is vector eq'n for the line of intersection of the two planes.

OR, simply:

$\langle 3t+1, -2t-5, t\rangle = \bar{r}(t)$

For $\bar{r}(t)$

203 T1

2 a 5 pts

$$\boxed{\bar{r}(t) = (1-t)\langle 3,-1,2\rangle + t\langle -3,2,1\rangle \quad \forall\, t \in [0,1]}$$ gives the

parametric eqns of line segment from $A(3,-1,2)$ to $B(-3,2,1)$

$\overrightarrow{AB} = \langle -3-3, 2+1, 1-2\rangle$

2b 5 pts $\overrightarrow{AB} = \boxed{\langle -6,3,-1\rangle = \bar{u}}$ Let

$\bar{r}(t) = \langle 3,-1,2\rangle + t\bar{u}$

Let $\boxed{\bar{r}_0 = \langle 3,-1,2\rangle} \Longrightarrow \boxed{\bar{r}(t) = \bar{r}_0 + t\bar{u}}$

OR $\boxed{\langle 3-6t, -1+3t, 2-t\rangle}$

~~$\langle 3, 3t-1, t+2\rangle$~~

2c 10 pts $\overrightarrow{AC} = \boxed{\langle 5,-4,2\rangle = \bar{v}}$

Then $\boxed{\bar{r}(t) = \bar{r}_0 + t\bar{u} + s\bar{v} \quad \forall\, (t,s) \in \mathbb{R}^2}$

$\langle 3,-1,2\rangle + t\langle -6,3,-1\rangle + s\langle 5,-4,2\rangle = \bar{r}$

$\langle 3-6t+5s, -1+3t-4s, 2+3t+2s\rangle$

:24 AM

203 T 1

2c DONE using cross product:

$\bar{u} = \langle\ \ , 3, -1\rangle$, $\bar{v} = \langle 5, -4, 2\rangle$

$\bar{n} = \bar{u} \times \bar{v}$: Initial Point $A = (3,-1,2)$
$= (x_0, y_0, z_0)$

10 pts

$$\begin{array}{r} \cancel{-6}, 3, -1, \cancel{-6}, 3 \\ \times\ 5, -4, 2, 5, -4 \\ \hline \langle 2, \cancel{7}, \ \ 9\rangle = \bar{n} \end{array}$$

$\bar{n} \cdot \bar{w} = 0$ if $\bar{w}$ in the plane.

$\bar{w} = \langle x-3, y+1, z-2\rangle$

$$\boxed{\bar{n} \cdot \bar{w} = 2(x-3) - 5(y+1) - 15(z-2) = 0}$$

If they didn't quite do the vector eq'n, which was actually easier than messing with cross product & standard eq'n (General Equation) of a plane!

203

J1

2d

10pts

Area $= \|\bar{u} \times \bar{v}\|$:

$2\overline{)254}$
127

$$\begin{array}{r} 6, 3, -1, 6, 3 \\ \times \; 5, -4, 2, 5, -4 \\ \hline \end{array}$$

$\langle 2, 7, 9 \rangle = \bar{u} \times \bar{v} \longrightarrow$

$\|\bar{u} \times \bar{v}\| = \sqrt{2^2 + 7^2 + 9^2} = \sqrt{4\,134}$

$\boxed{= \sqrt{134}}$ = Area of the ll-gram defined

by $\bar{u}$ & $\bar{v}$

203 $\overline{\mathcal{P}}$ 1

2e 10 pts Volume of parallelepiped:

$\vec{w} = \overrightarrow{AP} = \langle -5, -1, -1 \rangle$

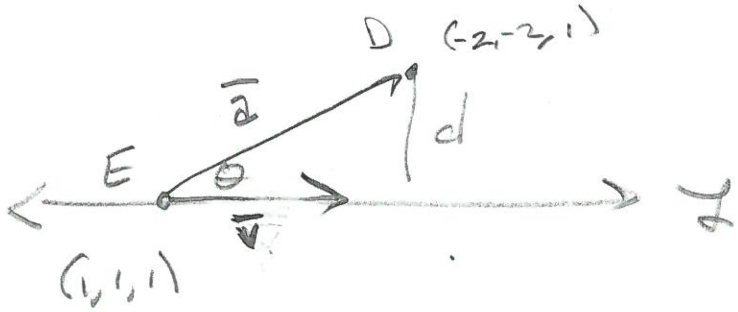
~~Ar~~ Volume $= | \bar{w} \cdot (\bar{u} \times \bar{v}) |$

$= | \langle -5, -1, -1 \rangle \cdot \langle 2, 7, 9 \rangle |$

$= | -10 - 7 - 9 | = \boxed{26 = \text{volume}}$

3 a $\bar{r}(t) = \langle 1, 1, 1 \rangle + t \langle 1, -2, 3 \rangle = \bar{r}_0 + t\bar{v}$

10 pts



$\boxed{\bar{a} = \langle -3, -3, 0 \rangle}$

from $\langle -2-1, -2-1, 1-1 \rangle$
$= \overrightarrow{ED}$

$\frac{d}{\|\bar{a}\|} = \sin\theta \Longrightarrow d = \|\bar{a}\| \sin\theta$

$= \|\bar{a}\| \frac{\|\bar{a} \times \bar{v}\|}{\|\bar{a}\| \, \|\bar{v}\|} = \frac{\|\bar{a} \times \bar{v}\|}{\|\bar{v}\|}$

$2|42$
$3|21$
7

$$\begin{array}{r} -3, -3, 0, -3, -3 \\ \times \quad 1, -2, 3, 1, -2 \\ \hline \langle -9, 9, 9 \rangle = \bar{a} \times \bar{v} \end{array}$$

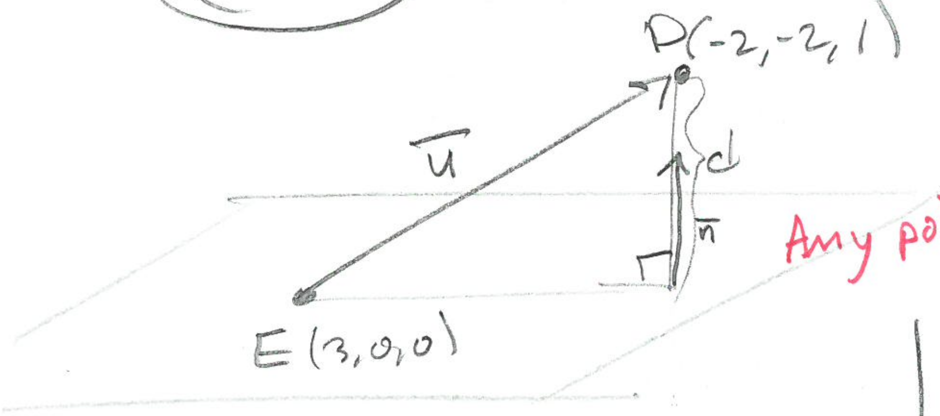
$\frac{\sqrt{3 \cdot 9^2}}{\sqrt{1^2 + 2^2 + 3^2}} = \boxed{\frac{9\sqrt{3}}{\sqrt{14}}}$ OR $\frac{9\sqrt{42}}{14}$

$\bar{a} \times \bar{v}$:

$$\begin{array}{r} -3, -3, 0, -3, -3 \\ \times \quad 1, -2, 3, 1, -2 \\ \hline \langle -9, 9, 9 \rangle \end{array} \longrightarrow \| \langle -9, 9, 9 \rangle \| = \sqrt{3 \cdot 9^2} = 9\sqrt{3}$$

203

3b 10 pts



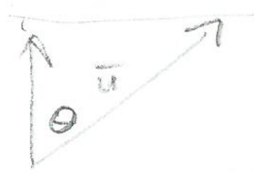
$\mathcal{P}: 2x-3y+5z=6$

Any point $E = \langle 3,0,0\rangle \in \mathcal{P}$

$$\boxed{\begin{aligned}\bar{n} &= \langle 2,-3,5\rangle \\ \bar{u} &= \langle -5,-2,1\rangle\end{aligned}}$$

$$d = |\text{comp}_{\bar{n}}\,\bar{u}| = \frac{|\bar{u}\cdot\bar{n}|}{\|\bar{n}\|}$$

$$= \frac{|-10+6+5|}{\sqrt{4+9+25}} = \boxed{\frac{1}{\sqrt{38}} = d}$$

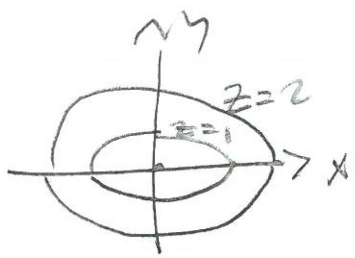


$\cos\theta =$

4 10 pts $9x^2-4z+25y^2=0$

$4z = 9x^2+25y^2$

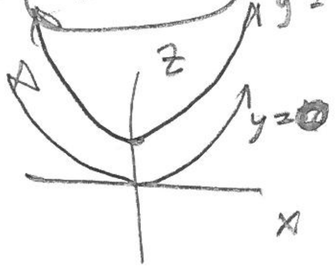
$z = \dfrac{x^2}{\frac{4}{9}} + \dfrac{y^2}{\frac{4}{25}} \overset{\text{SET}}{=} 1, 2, 3\ldots$ ellipses



xy

203

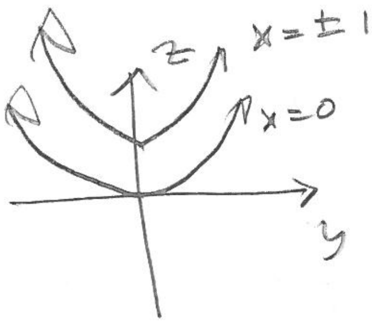
4 Cont'd



$y = \pm k$:

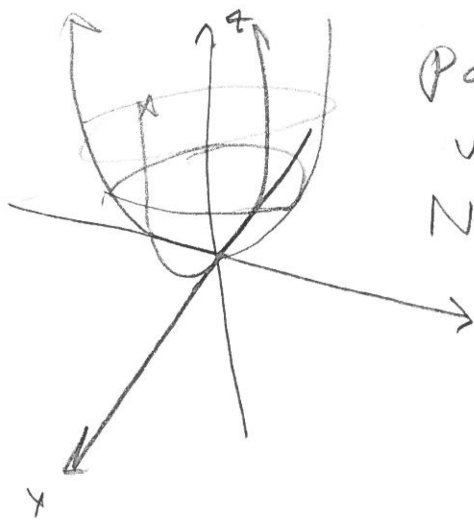
$$z = \frac{9}{4}x^2 + \frac{25}{4}k^2$$

Parabolas



$x = \pm k$

$$z = \frac{25}{4}y^2 + \frac{9}{4}k^2$$



Paraboloid

Wider in x-direction

Narrower in y-direction

by xy-projection

203 T1

5 $\bar{r}(t) = \langle \sin(t), t, \cos(t) \rangle$

a 5pts

$\Rightarrow r'(t) = \langle \cos(t), 1, -\sin(t) \rangle$

$\Rightarrow \| \bar{r}'(t) \| = \sqrt{\cos^2(t) + 1^2 + \sin^2(t)}$

$= \sqrt{2}$

$\Rightarrow \bar{T}(t) = \dfrac{\bar{r}'(t)}{\|\bar{r}'(t)\|} = \boxed{\dfrac{1}{\sqrt{2}} \langle \cos(t), 1, -\sin(t) \rangle = \bar{T}(t) = \langle \tfrac{1}{\sqrt{2}}\cos(t), \tfrac{1}{\sqrt{2}}, -\tfrac{1}{\sqrt{2}}\sin(t) \rangle}$

$\Rightarrow \bar{N} = \dfrac{\bar{T}'(t)}{\|\bar{T}'\|}$: $\bar{T}' = \dfrac{1}{\sqrt{2}} \langle -\sin(t)\ 0, -\cos(t) \rangle$

=

& $\|\bar{T}'\| = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \bar{N} = \dfrac{\frac{1}{\sqrt{2}} \langle -\sin(t), 0, -\cos(t) \rangle}{\frac{1}{\sqrt{2}}} = \boxed{\langle -\sin(t), 0, -\cos(t) \rangle = \bar{N}(t)}$

5 b 5pts $\bar{T}\left(\frac{\pi}{4}\right) = \dfrac{\langle \cos\frac{\pi}{4}, 1, -\sin\frac{\pi}{4} \rangle}{\sqrt{2}}$

$= \boxed{\dfrac{1}{\sqrt{2}} \langle \dfrac{1}{\sqrt{2}}, 1, -\dfrac{1}{\sqrt{2}} \rangle = \bar{T}\left(\frac{\pi}{4}\right)} = \langle \frac{1}{2}, \frac{1}{\sqrt{2}}, -\frac{1}{2} \rangle$

$\bar{N}\left(\frac{\pi}{4}\right) = \langle -\sin\frac{\pi}{4}, 0, -\cos\frac{\pi}{4} \rangle \boxed{= \langle -\dfrac{1}{\sqrt{2}}, 0, -\dfrac{1}{\sqrt{2}} \rangle = \bar{N}\left(\frac{\pi}{4}\right)}$

203 T1

5b contd $\overline{B}\left(\frac{\pi}{4}\right) = \overline{T}\left(\frac{\pi}{4}\right) \times \overline{N}\left(\frac{\pi}{4}\right)$:

$$\begin{array}{rccccc} \overline{T} & \frac{1}{2}, & \frac{1}{\sqrt{2}}, & -\frac{1}{2}, & \frac{1}{2}, & \frac{1}{\sqrt{2}} \\ \times & -\frac{1}{\sqrt{2}}, & 0, & -\frac{1}{\sqrt{2}}, & -\frac{1}{\sqrt{2}}, & 0 \\ \hline \end{array}$$

$$\left\langle -\frac{1}{2},\ \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}},\ -\frac{1}{2} \right\rangle$$

$$= \left\langle -\frac{1}{2}, \frac{1}{\sqrt{2}}, -\frac{1}{2} \right\rangle = \overline{B}\left(\frac{\pi}{4}\right)$$

5c 5pts

$\langle \sin t, t, \cos t \rangle$

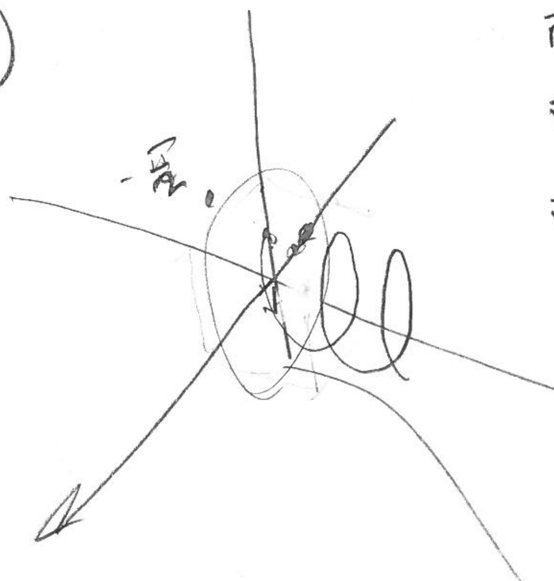
$\langle 0, 0, 1 \rangle$

$\left\langle \frac{1}{2}, \frac{\pi}{6}, \frac{\sqrt{3}}{2} \right\rangle$

$\left\langle \frac{1}{\sqrt{2}}, \frac{\pi}{4}, \frac{1}{\sqrt{2}} \right\rangle$

$\left\langle \frac{\sqrt{3}}{2}, \frac{\pi}{3}, \frac{1}{2} \right\rangle$

$\left\langle 1, \frac{\pi}{2}, 0 \right\rangle$



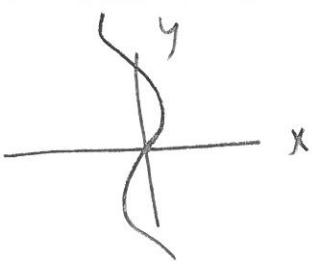
$\overline{r}\left(-\frac{\pi}{2}\right) = \left\langle -1, -\frac{\pi}{2}, 0 \right\rangle$

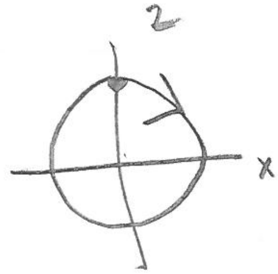
$\overline{r}\left(-\frac{\pi}{4}\right) = \left\langle -\frac{1}{\sqrt{2}}, -\frac{\pi}{4}, \frac{1}{\sqrt{2}} \right\rangle$

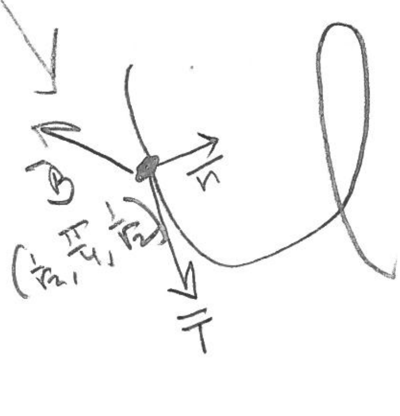
$\overline{r}(0) = \langle 0, 0, 1 \rangle$

$\overline{r}\left(\frac{\pi}{4}\right) = \left\langle \frac{1}{\sqrt{2}}, \frac{\pi}{4}, \frac{1}{\sqrt{2}} \right\rangle$

$\overline{r}\left(\frac{\pi}{2}\right) = \left\langle 1, \frac{\pi}{2}, 0 \right\rangle$







203 J 1

6 5pts $a_T = \dfrac{\bar{r}'(t) \cdot \bar{r}''(t)}{\|\bar{r}'(t)\|}$, $\bar{a}_N = \dfrac{\|\bar{r}'(t) \times \bar{r}''(t)\|}{\|\bar{r}'(t)\|}$

$\bar{r}(t) = \langle 1, t, t^2 \rangle \longrightarrow$

$\bar{r}'(t) = \langle 0, 1, 2t \rangle \longrightarrow$

$\bar{r}''(t) = \langle 0, 0, 2 \rangle \longrightarrow$

$\|\bar{r}'(t)\| = \sqrt{4t^2+1}$

$\bar{r}' \times \bar{r}''$: 0, 1, 2t, 0, 1

$\times$ 0, 0, 2, 0, 0

$\langle 2, 0, 0 \rangle = \bar{r}' \times \bar{r}''$

$\longrightarrow \boxed{\|\bar{r}' \times \bar{r}''\| = 2}$

$\boxed{\bar{r}' \cdot \bar{r}'' = 4t} \longrightarrow$

$\boxed{a_T = \dfrac{4t}{\sqrt{4t^2+1}}}$ & $\boxed{a_N = \dfrac{2}{\sqrt{4t^2+1}}}$

Check:

$\bar{a} = \bar{a}_T \bar{T} + a_N \bar{N}$

203 J1

7 5pts $\kappa = \left\| \frac{dT}{ds} \right\|$ is the curvature, which is given by the rate of change in the unit ~~vect~~ tangent with respect to arc length. It tells us how sharp the curve is turning.

$$\boxed{\kappa = \frac{\|\bar{r}'(t) \times \bar{r}''(t)\|}{\|\bar{r}'(t)\|^3}}$$

8 $$\frac{d}{dx}\int_0^{6x^2+1} \frac{\sin^2(3\tau) + \tau^5}{\sqrt{6\tau^3 + \cos^2(3\tau)}}\, d\tau$$

$$\boxed{= \left(\frac{\sin^2(3(6x^2+1)) + (6x^2+1)^5}{\sqrt{6(6x^2+1) + \cos^2(3(6x^2+1))}}\right)(12x)}$$

203 J1

9 5 pts $\bar{v}(0) = \langle 500\cos 45^\circ, 500\sin 45^\circ \rangle$

$\bar{r}(t) = \bar{v}(0)t + \langle 0, -9.8t^2 \rangle$

$= \langle \frac{500}{\sqrt{2}}t, \frac{500}{\sqrt{2}}t - 9.8t^2 \rangle + \bar{r}_0$ $= \bar{0}$

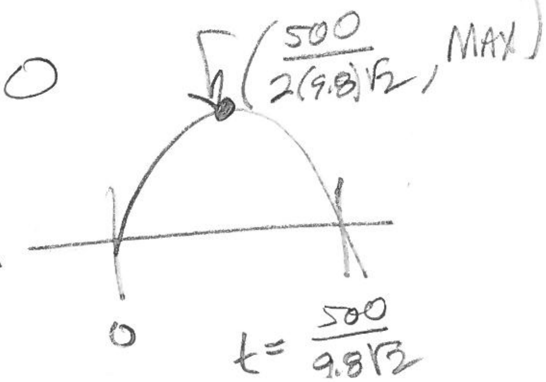
y-coord controls max height & time to ground

$y = \frac{500}{\sqrt{2}}t - 9.8t^2 \stackrel{\text{SET}}{=} 0$

$\longrightarrow 9.8t^2 - \frac{500}{\sqrt{2}}t = t\left(9.8t - \frac{500}{\sqrt{2}}\right) = 0$

$\longrightarrow t = 0$ OR $9.8t - \frac{500}{\sqrt{2}} = 0$

When it hits the ground: $\longrightarrow \boxed{t = \frac{500}{9.8\sqrt{2}}}$



RANGE: $\left(\frac{500}{\sqrt{2}}\right)\left(\frac{500}{9.8\sqrt{2}}\right) = x$

$= \frac{500^2}{(9.8)(2)} = \boxed{x \approx 12755.10204\,m}$ $\approx$ RANGE

MAX HEIGHT: $\left(\frac{500}{\sqrt{2}}\right)\left(\frac{500}{2(9.8)\sqrt{2}}\right) - 9.8\left(\frac{500}{2(9.8)\sqrt{2}}\right)^2$

SPEED WHEN IT HITS GROUND! $= \sqrt{\left(\frac{500}{\sqrt{2}}\right)^2 + \left(\frac{500}{\sqrt{2}}\right)^2} = \sqrt{\frac{2(500)^2}{2}}$ $\boxed{= 500\,\frac{m}{s}! \text{ speed of impact}}$

203 T1

(10) $\bar{a}(t) = \langle \sin t, t, 3t+1 \rangle$

$\bar{r}(0) = \langle 1,1,1 \rangle$, $\bar{v}(0) = \langle -1,2,0 \rangle$ ← TYPO on test

$\Longrightarrow$

$\bar{v}(t) = \langle -\cos t, \frac{t^2}{2}, \frac{3}{2}t^2 + t \rangle + \bar{c}$

~~$= \langle \cos t, \frac{t^2}{2}, \frac{3}{2}t^2 + t \rangle + \langle -1,2,0 \rangle$~~

~~$= \langle \cos t - 1, \frac{1}{2}t^2 + 2, \frac{3}{2}t^2 + t \rangle$~~

~~$\Longrightarrow$ $\bar{r}(t) = \langle -\sin t - t, \frac{1}{6}t^3 + 2t, \frac{3}{6}t^3 + \frac{1}{2}t^2 \rangle + \bar{r}(0)$~~

$\bar{v}(0) = \langle -1,2,0 \rangle = \langle -\cos(0), 0, 0 \rangle + \bar{c}$

$= \langle -1,0,0 \rangle + \bar{c}$

$\longrightarrow \bar{c} = \langle 0,2,0 \rangle \longrightarrow \bar{v}(t) = \langle -\cos t, \frac{1}{2}t^2 + 2, \frac{3}{2}t^2 + t \rangle$

$\Longrightarrow \bar{r}(t) = \langle -\sin t, \frac{1}{6}t^3 + 2t, \frac{3}{6}t^3 + \frac{1}{2}t^2 \rangle + \bar{d}$

$\bar{r}(0) = \langle 0,0,0 \rangle + \bar{d} = \langle 1,1,1 \rangle \longrightarrow \bar{d} = \langle 1,1,1 \rangle$

$\Longrightarrow \boxed{\bar{r}(t) = \langle -\sin t + 1, \frac{1}{6}t^3 + 2t + 1, \frac{1}{2}t^3 + \frac{1}{2}t^2 + 1 \rangle}$