MAT 203 Test 1 – Spring, 2019 Name_________________

100 Points Covers Chapters 12, 13 NO GRAPHING CALCULATORS!!!

Do all your work and submit answers with your work, on the separate paper provided. Organize your work for efficient grading and feedback. Leave a margin, especially in the top left, where the staple goes!

Leave space between problems. No prizes for saving paper, here. Figure this stuff out, and use your smarts to plant trees! Only use one column of work. Don't start a 2nd column to save paper. ALL I WANT ON THIS PAGE IS YOUR NAME.

1. (10 pts) Find parametric equations and a vector equation for the line of intersection between the two planes:
   $P_1:\ x+y-z=-4$
   $P_2:\ 2x+y-4z=-3$

2. Essential concepts for math in 3-D: We'll be working with the 4 points $A(3,-1,2)$, $B(-3,2,1)$, $C(8,-5,4)$ and $D(-2,-2,1)$.

   a. (5 pts) (Line segment) Write the equation of the line segment between $A(3,-1,2)$ and $B(-3,2,1)$.

   b. (5 pts) (Line) Form the vector $\bar{u}=\overrightarrow{AB}$ and find a vector equation for the line containing the points $A(3,-1,2)$ and $B(-3,2,1)$.

   c. (10 pts) (Vector Equation of Plane) Form the vector $\bar{v}=\overrightarrow{AC}$, using *A* from part a. and $C(8,-5,4)$. Then write a vector equation for the plane containing *A*, *B* and *C*. Sketch the plane using its intercepts.

   d. (10 pts) (Area of Parallelogram) Find the area of the parallelogram defined by the vectors $\bar{u}$ and $\bar{v}$.

   e. (10 pts) (Volume of a Parallelepiped) Form the vector $\bar{w}=\overrightarrow{AD}$, where $D(-2,-2,1)$ is another point. And find the volume of the parallelepiped defined by the 3 edges, $\bar{u},\bar{v}$, and $\bar{w}$.

3. Distance Problems:

   a. (10 pts) (Distance from a Point to a Line) Let $\bar{r}(0)=\bar{r}_0=\langle 1,1,1\rangle$ and $\bar{v}=\langle 1,-2,3\rangle$. Find the distance from the point $D(-2,-2,1)$ to the line L: $\bar{r}(t)=\bar{r}_0+t\bar{v}$. This is not the same $\bar{v}$ as in #2. For convenience, let $E(1,1,1)$ correspond for the initial position vector $\bar{r}_0=\langle 1,1,1\rangle$.

   b. (10 pts) (Distance from a Point to a Plane) Find the distance from the point $D(-2,-2,1)$ to the plane P: $P: 2x-3y+5z=6$

4. (10 pts) Consider the quadric surface $9x^2-4z+25y^2=0$. Show its traces in the planes $x=k, y=k, z=k$ for different choices (at least 2 each) of *k* and project those into the $yz-, xz-,$ and $xy-$ planes, respectively. In other words, sketch the projections into the $yz-, xz-,$ and $xy-$ planes. Then sketch the surface in 3-D.

5. Let $\bar{r}(t)=\langle \sin(t), t, \cos(t)\rangle$.

   a. (5 pts) Find the Unit Tangent $\bar{T}$, Unit Normal $\bar{N}$, as functions of t.

b. (5 pts) Evaluate your answers to part a at $t=\frac{\pi}{4}$ *and* find the Unit Binormal $\overline{B}$ at $t=\frac{\pi}{4}$. In other words, find $\overline{T}\left(\frac{\pi}{4}\right), \overline{N}\left(\frac{\pi}{4}\right)$, and $\overline{B}\left(\frac{\pi}{4}\right)$.

c. (5 pts) Sketch the graph of $\overline{r}(t)$, for $t\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. Then draw the TNB frame that you found symbolically in part b. That is, show the unit tangent, normal and binormal in your sketch.

6. (5 pts) Find the tangential and normal components of the acceleration vector for the vector function $\overline{r}(t)=\langle 1,t,t^2\rangle$.

BONUS: Answer up to 3 for up to 15 points.

7. (5 pts) Give a verbal description of the statement $\kappa=\left|\frac{d\overline{T}}{ds}\right|$. What is it? What does it mean? What's our shortcut for calculating it, in terms of $\overline{r}(t)$?

8. (5 pts) Simplify the derivative: : $\frac{d}{dx}\int_0^{6x^2+1}\frac{\sin^2(3\tau)+\tau^5}{\left(\sqrt{6\tau^3}+\cos^2(3\tau)\right)}d\tau$

9. (5 pts) A projectile is fired with an initial velocity of 500 m/s and an angle of elevation of 45^0. Find its maximum height, range, and speed at which it strikes the ground. And no, our model doesn't work on a spinning planet with spinning projectile and air resistance. It's actually quite complicated. But we'd do great with this stuff on the moon.

10. Find the position $\overline{r}(t)$, velocity $\overline{v}(t)$, given the acceleration $\overline{a}(t)=\langle\sin(t),t,3t+1\rangle$, and given $\overline{r}(0)=\langle 1,1,1\rangle$ and $\overline{v}(t)=\langle -1,2,0\rangle$.