7–10 Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$. In each case C is oriented counterclockwise as viewed from above.

7. $\mathbf{F}(x, y, z) = (x + y^2)\,\mathbf{i} + (y + z^2)\,\mathbf{j} + (z + x^2)\,\mathbf{k}$,
C is the triangle with vertices (1, 0, 0), (0, 1, 0), and (0, 0, 1)

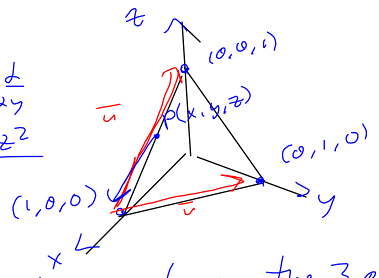
c_1 c_2 c_3

$$\int_C \bar{F}\cdot d\bar{r} = \sum_{k=1}^{3} \int_{C_k} \bar{F}\cdot d\bar{r}$$

$\text{curl}\,\bar{F} = \nabla \times \bar{F}$

$$\begin{array}{ccc|cc} \frac{\partial}{\partial x}, & \frac{\partial}{\partial y}, & \frac{\partial}{\partial z} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ x+y^2, & y+z^2, & z+x^2 & x+y^2, & y+z^2 \\ \hline \end{array}$$

$\langle 0-2z,\ 0-2x,\ 0-2y\rangle$



$\iint_S \text{curl}\,\bar{F}\cdot d\bar{S}$ $\quad S$ is the plane containing the 3 pts.

The plane's description:

linear combo of $\bar{u}$ & $\bar{v}$.

one way:

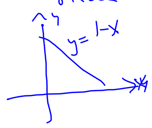
$\bar{r}(s,t) = \bar{r}_0 + s\bar{u} + t\bar{v}$

$= \langle 1,0,0\rangle + s\langle -1,0,1\rangle + t\langle -1,1,0\rangle$

$= \langle 1-s-t,\ t,\ s\rangle$

$\bar{u} = \langle -1, 0, 1\rangle, -1, 0$
$\times\ \bar{v} = \langle -1, 1, 0\rangle, -1, 1$

$\langle -1,-1,-1\rangle$ = one $\bar{n}$ we could use, but we want $\langle 1,1,1\rangle = \bar{n}$, the "outward" normal, from standard counterclockwise orientation of C.



$$\iint_S \text{curl}\,\bar{F}\cdot d\bar{S} = \iint \text{curl}\,\bar{F}\cdot\bar{n}\,dS$$

$$= \iint \text{curl}\,\bar{F}\cdot(\bar{r}_x \times \bar{r}_y)\,dy\,dx$$

$$= \int_0^1\int_0^{1-x} \langle -2z,-2x,-2y\rangle\cdot\langle 1,1,1\rangle\,dy\,dx$$

$\overline{\|\bar{r}_x\times\bar{r}_y\|}$

$\|\bar{r}_x\times\bar{r}_y\|$

$$= -2\int_0^1\int_0^{1-x}(z+x+y)\,dy\,dx$$

$$= -2\int_0^1\int_0^{1-x}((1-x-y)+x+y)\,dy\,dx$$

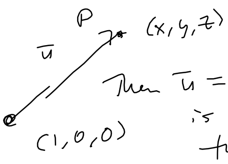
$$= -2\int_0^1\int_0^{1-x} dy\,dx$$

$$= -2\int_0^1 (1-x)\,dx$$

$$= +2\left[\frac{(1-x)^2}{2}\right]_0^1$$

$$= \frac{2}{2}\left[0-1\right] = -1$$

Another way to build the plane. We have $\bar{n} = \langle 1,1,1\rangle$ is $\perp$ to $\mathcal{P}$ & $(1,0,0)$ is in $\mathcal{P}$.
Let $P = (x,y,z) \in \mathcal{P}$



Then $\bar{u} = \langle x-1, y, z\rangle$ is a vector in the plane!

$\therefore\ \bar{n}\cdot\bar{u} = 0$

$\langle 1,1,1\rangle\cdot\langle x-1,y,z\rangle =$

$x-1+y+z = 0$

$x+y+z = 1$

$z = 1-x-y$

5–15 Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$; that is, calculate the flux of $\mathbf{F}$ across S.

5. $\mathbf{F}(x, y, z) = e^x \sin y\, \mathbf{i} + e^x \cos y\, \mathbf{j} + yz^2\, \mathbf{k}$,
S is the surface of the box bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 2$

$\nabla \cdot \vec{F}$ $\quad$ $\text{Div } \vec{F} = e^x \sin y + (-e^x \sin y) + 2yz \quad = 2z$

$$\int_0^2 \int_0^1 \int_0^1 2yz \; dy\, dx\, dz =$$

$$= 2 \int_0^2 z\, dz \int_0^1 d\cancel{x} \int_0^1 y\, dy$$

$$= 2 \left[\frac{z^2}{2}\right]_0^2 \left[x\right]_0^1 \left[\frac{y^2}{2}\right]_0^1$$

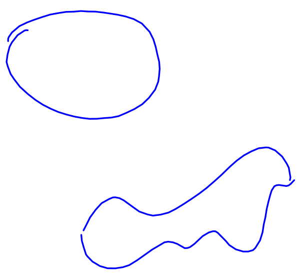
$$= 2 \, [2][1]\left[\frac{1}{2}\right] = 2$$

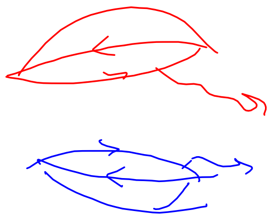
Sphere is 2 hemispheres.

STOKES'

$$\int_C \bar{F} \cdot d\bar{r} = \boxed{\iint_S \text{curl}\,\bar{F} \cdot d\bar{S} = \iint_{\text{TOP half}}} + \iint_{\text{BOTTOM HALF}}$$

Break the sphere into upper & lower hemispheres.





C_1

$-C_1 = C_2$