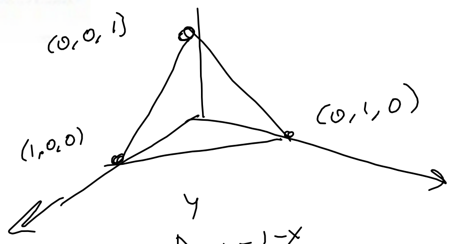
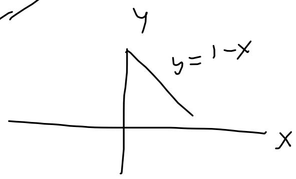
5–18 Evaluate the surface integral.

7. $\iint_S yz\, dS$, S is the part of the plane $x + y + z = 1$ that lies in the first octant

$dS = \|\bar{r}_x \times \bar{r}_y\| \, dy\, dx$





$z = 1 - x - y$

$\bar{r} = \langle x, y, 1-x-y \rangle$

$\bar{r}_x = \langle 1, 0, -1 \rangle$ 1, 0

$\bar{r}_y = \langle 0, 1, -1 \rangle$, 0, 1

$\langle 1, 1, 1 \rangle = \bar{r}_x \times \bar{r}_y$

$\|\bar{r}_x \times \bar{r}_y\| = \sqrt{3}$

$$\iint_S yz\, dS = \int_0^1 \int_0^{1-x} y(1-x-y)\sqrt{3}\, dy\, dx = \frac{\sqrt{3}}{24}$$

$$= \sqrt{3}\int_0^1 \int_0^{1-x} (y - xy - y^2)\, dy\, dx = \sqrt{3}\int_0^1 \left[(1-x)\frac{y^2}{2} - \frac{1}{3}y^3 \right]_0^{1-x} dx$$

Scratch: $\frac{(1-x)(1-x)^2}{2} - \frac{1}{3}(1-x)^3 = \frac{3[1-x]^3 - 2[1-x]^3}{6}$

$= \frac{1}{6}[1-x]^3 = \frac{1}{6}[1 - 3x + 3x^2 - x^3] =$

$$\frac{\sqrt{3}}{6}\int_0^1 [1 - 3x + 3x^2 - x^3]\, dx = \frac{\sqrt{3}}{6}\left[x - \frac{3x^2}{2} + x^3 - \frac{1}{4}x^4 \right]_0^1$$

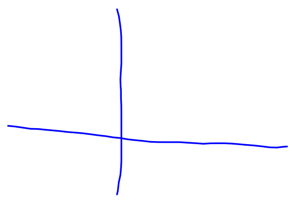
$$= \frac{\sqrt{3}}{6}\left[1 - \frac{3}{2} + 1 - \frac{1}{4}\right] = \frac{\sqrt{3}}{6}\left[\frac{8-6-1}{4}\right]$$

$$= \frac{\sqrt{3}}{6}\left[\frac{1}{4}\right] = \frac{\sqrt{3}}{24}$$

17. $x = \cos^3 u \cos^3 v, \quad y = \sin^3 u \cos^3 v, \quad z = \sin^3 v$

u = constant

$\langle K_1 \cos^3 v, K_2 \cos^3 v, \sin^3 v \rangle$



Go to 16-7 video

https://harryzaims.com/203/videos/chapter-16/16-4/16-4-num-12-notes.pdf

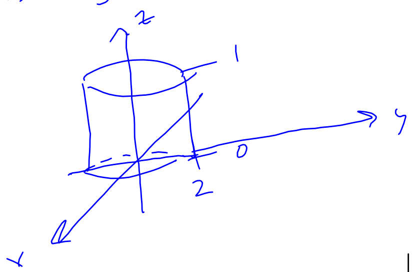
The video that goes with it will walk you through the #12 with Maple.

41. A fluid has density 870 kg/m^3 and flows with velocity $\mathbf{v} = z\,\mathbf{i} + y^2\,\mathbf{j} + x^2\,\mathbf{k}$, where x, y, and z are measured in meters and the components of $\mathbf{v}$ in meters per second. Find the rate of flow outward through the cylinder $x^2 + y^2 = 4$, $0 \leq z \leq 1$.

$\bar{v} = \langle z, y^2, x^2 \rangle$ in m/s

x, y, z in m

$\rho(x,y,z) = 870 \frac{kg}{m^3}$



$\rho \iint_S \bar{v} \cdot d\bar{S} =$

$\rho \iint_S \bar{v} \cdot \bar{n}\, dS$

$\bar{n}$ for the sides is harder

$= \rho \iint_S \bar{v} \cdot \bar{r}_\theta \times \bar{r}_z \, d\theta dz$

Top: $\langle 0,0,1 \rangle$

Bottom: $\langle 0,0,-1 \rangle$

Sides $r = 2$, cylindricals

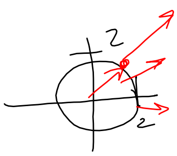
$\bar{r} = \langle 2\cos\theta, 2\sin\theta, z \rangle$ $\quad 0 \le \theta \le 2\pi$, $0 \le z \le 1$

$\bar{r}_\theta = \langle -2\sin\theta, 2\cos\theta, 0 \rangle$ $-2\sin\theta, 2\cos\theta$

$\times \quad \bar{r}_z = \langle 0, 0, 1 \rangle$ $\quad 0, 0$

$\langle 2\cos\theta, 2\sin\theta, 0 \rangle = \bar{r}_\theta \times \bar{r}_z$ is a normal.

$\int_0^{2\pi} \int_0^1$



$d\bar{S} = \bar{n}\, dS$

$\frac{\bar{r}_\theta \times \bar{r}_z}{\|\bar{r}_\theta \times \bar{r}_z\|}$ ~~$\|\bar{r}_\theta \times \bar{r}_z\|$~~ $dz\, d\theta$

So $\rho \iint_S \bar{v} \cdot d\bar{S} = \rho \left[\iint_{S_1} + \iint_{S_2} + \iint_{S_3} \right]$

S_1: Bottom, S_2: Top, S_3: side

I_1: $\rho \int_0^{2\pi} \int_0^2 \langle 0, r^2\sin^2\theta, r^2\cos^2\theta \rangle \cdot \langle 0,0,-1 \rangle \, r\,dr\,d\theta$

$\bar{r} = \langle r\cos\theta, r\sin\theta, 0 \rangle$

$dS = r\,dr\,d\theta$

$\bar{v} = \langle z, y^2, x^2 \rangle$

$= \langle 0, r^2\sin^2\theta, r^2\cos^2\theta \rangle$

$\bar{v} \cdot \bar{n}\, dS =$

$I_2 = -\rho \int_0^{2\pi} \int_0^2$ ~~

So $I_1 + I_2 = 0$!

9. $\iint_S yz\,dS$,
S is the surface with parametric equations $x = u^2$, $y = u \sin v$,
$z = u \cos v$, $0 \leqslant u \leqslant 1$, $0 \leqslant v \leqslant \pi/2$

14. $\iint_S y^2\, dS$,
S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane

17. $\iint_S (z + x^2y)\, dS$,
S is the part of the cylinder $y^2 + z^2 = 1$ that lies between the planes $x = 0$ and $x = 3$ in the first octant

17. $\iint_S (z + x^2 y)\, dS$,
S is the part of the cylinder $y^2 + z^2 = 1$ that lies between the planes $x = 0$ and $x = 3$ in the first octant