§15.9

$$3x-2y=12$$
$$5x+y=10$$

$3x-2y=u$

$5x+y=v$

$y=v-5x$

$3x-2(v-5x)=u$

$3x-2v+10x=u$

$13x=u+2v$

$\boxed{x=\frac{u+2v}{13}}$

$y=v-5x$

$=v-5\left(\frac{u+2v}{13}\right)$

$=v-\frac{5}{13}(u+2v)$

$=v-\frac{5}{13}u-\frac{10}{13}v$

$\boxed{y=-\frac{5}{13}u+\frac{3}{13}v}$

$\vec{r}=\frac{1}{13}\langle u+2v,-5u+3v\rangle$

$\vec{r}_u=\frac{1}{13}\langle 1,-5\rangle$

$\vec{r}_v=\frac{1}{13}\langle 2,3\rangle$

$$\left|\begin{vmatrix}\frac{1}{13} & -\frac{5}{13}\\ \frac{2}{13} & \frac{3}{13}\end{vmatrix}\right|=\frac{1}{13}\begin{vmatrix}1 & -5\\ \frac{2}{13} & \frac{3}{13}\end{vmatrix}$$

$$=\frac{1}{169}\begin{vmatrix}1 & -5\\ 2 & 3\end{vmatrix}=\frac{1}{169}(3+10)=\frac{1}{169}(13)=\frac{1}{13}$$

$$3x-2y=12$$
$$5x+y=10$$

$\begin{vmatrix}3 & -2\\ 5 & 1\end{vmatrix}=3+10=13$, so

$\left|\frac{\partial(x,y)}{\partial(u,v)}\right|=\frac{1}{13}$!

$$\begin{aligned} x+2y+2z &= \cancel{3}\ u \\ 2x-y+7z &= \cancel{4}\ v \\ -x+3y-5z &= \cancel{-1}\ w \end{aligned}$$

This matrix is not invertible, Mills.

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with(LinearAlgebra):

A = <<1,2,-1>|<2,-1,3>|<2,7,-5>|<u,v,w>>
    ReducedRowEchelonForm(A)
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$$x+2y+2z=7$$
$$-2x+y+4z=0$$
$$2y+3z=6$$

$$\begin{bmatrix} 1 & 2 & 2 & u \\ -2 & 1 & 4 & v \\ 0 & 2 & 3 & w \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & -6w+2v+5u \\ 0 & 1 & 0 & 8w-3v-6u \\ 0 & 0 & 1 & -5w+2v+4u \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & \sim & \sim & \sim \\ 0 & 1 & \sim & \sim \\ 0 & 0 & \sim & \sim \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & u \\ -2 & 1 & 4 & v \\ 0 & 2 & 3 & w \end{array}\right] \begin{array}{c} R1 \\ 2R1+R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 2 & u \\ 0 & 5 & 8 & 2u+v \\ 0 & 2 & 3 & w \end{array}\right]$$

$$\begin{array}{c} R1 \\ R2 \\ -2R2+5R3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 2 & u \\ 0 & 5 & 8 & 2u+v \\ 0 & 0 & -1 & -4u-2v+5w \end{array}\right]$$

$x+2y+2z=4$

$5y+8z=2u+v$

$\boxed{z=4u+2v-5w}$

SCRATCH:

$-2R2$	0	-10	-16	$-4u-2v$
$5R3$	0	10	15	$5w$
$-2R2+5R3$	0	0	-1	$-4u-2v+5w$

$5y+8z=5y+8(4u+2v-5w)=2u+v$ Solve for y:

$5y+32u+16v-40w=2u+v$

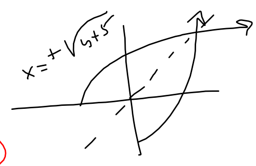
$5y=-30u-15v+40w$

$\boxed{y=-6u-3v+8w}$ ... $\boxed{5u+2v-6w=x}$

$\vec{r}=\langle 5u+2v-6w, \text{etc...} \rangle$

$y=x^2-5$, $\Rightarrow x^2=y+5$

$\Rightarrow x=\pm\sqrt{y+5}$



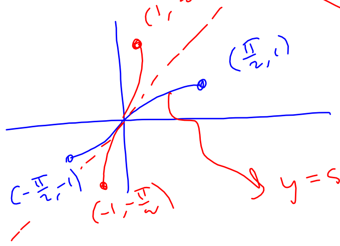
Right half of x^2?

$+\sqrt{}$

$x=\frac{\pi}{2}$

$x=\arcsin(y)$

So, $y=\sin(x)$!



$y=\sin(x)$ IS $x=\arcsin(y)$