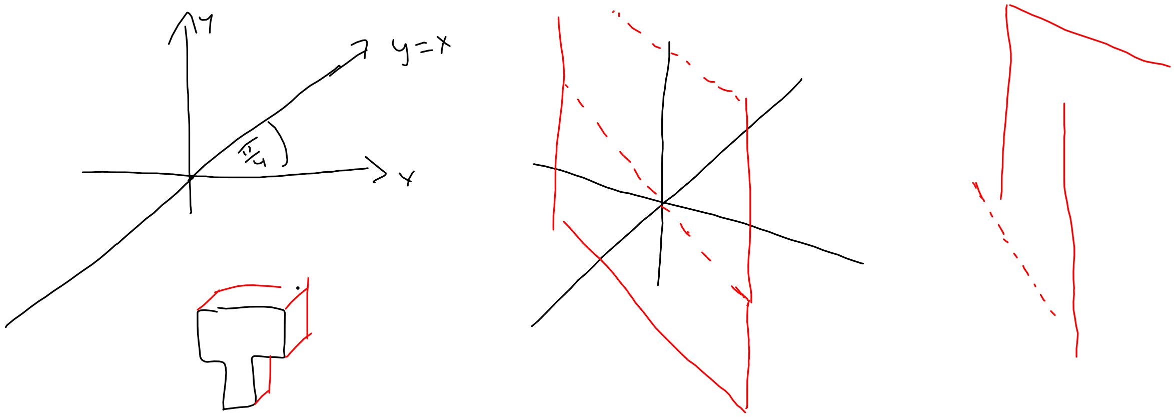
$\theta = \frac{\pi}{4}$ in cylindrical coordinates is...



If $x = u - v$, $y = u + v$ ..

Then $\bar{r}(u,v) = \langle u-v, u+v \rangle$

The tough part is getting the $\bar{r}$ for the

2-D $dA = \|\bar{r}_u \times \bar{r}_v\|$ &

3-D $dV = \|\bar{r}_u \cdot (\bar{r}_v \times \bar{r}_w)\|$

$\bar{r}_u = \langle 1, 1 \rangle$, $\bar{r}_v = \langle 1, -1 \rangle$

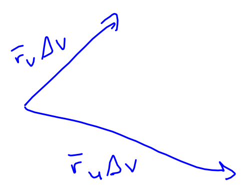
$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1-1 = -2$$

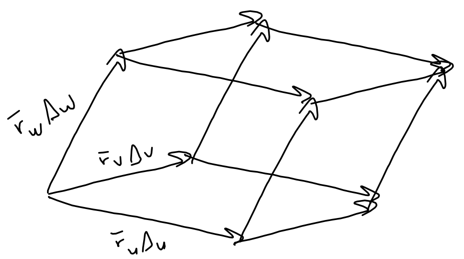
$$\text{so } dV = \left| \frac{\partial(x,y)}{\partial(u,v)} \right| = 2$$

$$\text{So, } \iint_R f\,dV = \iint_R f\,dy\,dx = \iint_R f\,dx\,dy$$

$$= \iint_S f(x(u,v), y(u,v)) \cdot 2\,du\,dv$$

$\|\bar{r}_u \times \bar{r}_v\|\,du\,dv$





$$\begin{vmatrix} x_u & y_u & z_u \\ x_v & y_v & z_v \\ x_w & y_v & z_v \end{vmatrix} \Delta u \Delta v \Delta w$$

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4-6 = -2$$

$$\begin{vmatrix} 5 & 10 \\ 3 & 4 \end{vmatrix} = 20-30 = -10 = 5 \cdot \text{previous.}$$

$$\langle x_u, y_u, z_u \rangle \cdot \left(\langle x_v, y_v, z_v \rangle \times \langle x_w, y_w, z_w \rangle \right)$$

Spherical Jacobian

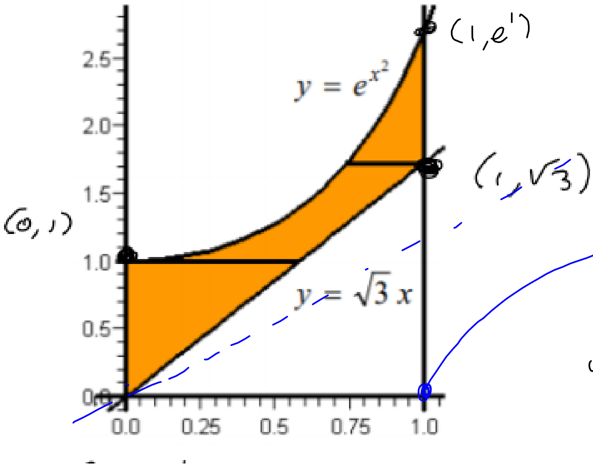
$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\phi$$

$$\bar{r} = \langle x, y, z \rangle = \langle \rho \sin\phi\cos\theta, \rho \sin\phi \sin\theta, \rho\cos\phi \rangle$$

$$\begin{vmatrix} \bar{r}_\rho \\ \bar{r}_\theta \\ \bar{r}_\phi \end{vmatrix} = \begin{vmatrix} \sin\phi\cos\theta & \sin\phi\sin\theta & \cos\phi \\ -\rho\sin\phi\sin\theta & \rho\sin\phi\cos\theta & 0 \\ \rho\cos\phi\cos\theta & +\rho\cos\phi\sin\theta & -\rho\sin\phi \end{vmatrix}$$

$$= \cos\phi\left[-\rho^2 \sin\phi\cos\phi\sin^2\theta - \rho^2\sin\phi\cos\phi\cos^2\theta\right]$$
$$- 0 \left[\quad \right]$$
$$+ -\rho\sin\phi\left[\rho\sin^2\phi\cos^2\theta + \rho\sin^2\phi\sin^2\theta\right]$$

You can see how messy it is!



TI: $y = x$

$$\int_0^1 \int_{\sqrt{3}\,x}^{e^{x^2}} f(x,y)\,dy\,dx$$

TII:

$y = \sqrt{3}\,x$

$x = \frac{1}{\sqrt{3}}\,y$

$y = e^{x^2}$

$\ln(y) = x^2$

$x = \pm\sqrt{\ln(y)}$

$x = \sqrt{\ln(y)}$

$$\int_0^1 \int_0^{\frac{1}{\sqrt{3}}y} dx\,dy + \int_1^{\sqrt{3}} \int_{\sqrt{\ln(y)}}^{\frac{1}{\sqrt{3}}y} dx\,dy + \int_{\sqrt{3}}^{e} \int_{\sqrt{\ln(y)}}^{1} dx\,dy$$

$(1-t)\bar{r}_0 + t\,\bar{r}_1$ line segment