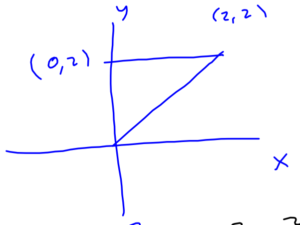
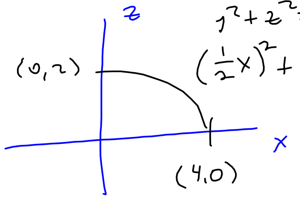
15.3 is where it starts getting interesting. This one's a bit tough for in-class, but good for take-home:

26. Bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x = 2y$, $x = 0$, $z = 0$ in the first octant

$y^2 + z^2 = 4$ $\quad x = 2y \quad$ $\cancel{\&}$ $\quad x = 0,$ $\cancel{\&}$ $z = 0$

$\langle 2y, y, \sqrt{4-y^2} \rangle$



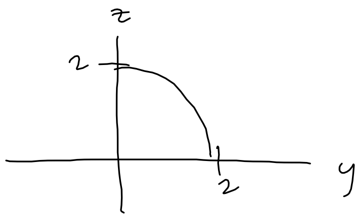


$y^2 + z^2 = 4$

$\left(\frac{1}{2}x\right)^2 + z^2 = 4$

$\frac{x^2}{2^2} + z^2 = 4$

$\frac{x^2}{4^2} + \frac{z^2}{2^2} = 1$



This is probably too easy.

12. $\displaystyle\iint_D x\sqrt{y^2 - x^2}\, dA, \quad D = \{(x, y) \mid 0 \leqslant y \leqslant 1,\ 0 \leqslant x \leqslant y\}$

This is probably Baby Bear's Porridge for difficulty (set-up only on sit-down portion).

19–28 Find the volume of the given solid.

20. Under the surface $z = 2x + y^2$ and above the region bounded by $x = y^2$ and $x = y^3$

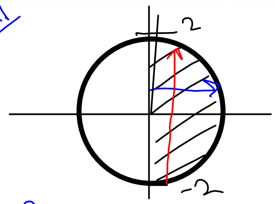
TEST 3 on MONDAY

Paraboloids fair game. Planes fair game. Spheres, Ellipsoids. But nothing too hairy.

15.4: Polar Coordinates.

10. $\iint_R \sqrt{4 - x^2 - y^2}\, dA$, where $R = \{(x, y) \mid \boxed{x^2 + y^2 \leq 4,}\ x \geq 0\}$ $= \iint_R \sqrt{4-r^2}$

TYPE II



Rectangular

$$T\text{II} \quad \int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \sqrt{4-x^2-y^2}\, dx\, dy$$

$$T\text{I} \quad \int_{0}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \sqrt{4-x^2-y^2}\, dy\, dx$$

$dA = r\, dr\, \theta$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^2 \boxed{\sqrt{4-r^2}\, r}\, dr\, d\theta$$

Even

$\to$ ODD. So what? That's got nothing to do with θ.

$$\cancel{= 2\int_0^{\frac{\pi}{2}} \int_0^2 \sqrt{4-r^2}\, r\, dr\, d\theta}$$

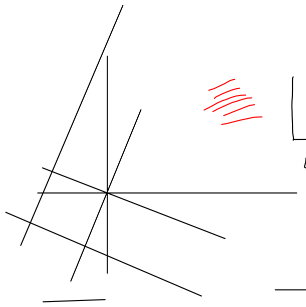
CRAP

$$\int_{-a}^{a} f(r)\, dr = \begin{cases} 0 & \text{if } f \text{ odd} \\ 2\int_0^a & \text{if } f \text{ even} \end{cases}$$

15.5 Applications - Not emphasized on Test 3.

15.6 Triple Integrals. Set up a Type 1 over a fairly simple Type I or Type II

15.9 Question I'd like to ask:



$y = 3x$

$y = 3x + 3$

$\boxed{\begin{array}{l} -3x+y=0 \\ -3x+y=3 \end{array}}$

$y = -\frac{1}{3}x$ $\boxed{x + 3y = 0}$

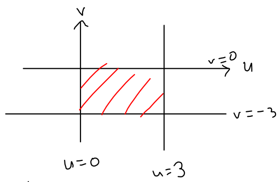
$y = -\frac{1}{3}x - \frac{3}{3}$

$3y = -x - 3$

$\boxed{x + 3y = -3}$

Let $u = -3x+y$

& $v = x+3y$



$$\int_{-3}^{0}\int_{0}^{3} du\,dv$$

Find Jacobian & Re-write.

Need $x = x(u,v)$ & $y = y(u,v)$

We have $u = u(x,y)$ & $v = v(x,y)$

$u = -3x + y$
$v = x + 3y$

$$T\bar{r} = \underbrace{\begin{bmatrix} -3 & 1 \\ 1 & 3 \end{bmatrix}}_{T}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix} =$$

Want $T^{-1}\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\left[\begin{array}{cc|c} -3 & 1 & u \\ 1 & 3 & v \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 3 & v \\ -3 & 1 & u \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 3 & v \\ 0 & 10 & u+3v \end{array}\right]$$

$x + 3y = v$

$10y = u + 3v$

$\boxed{y = \frac{1}{10}u + \frac{3}{10}v}$

$x + 3\left(\frac{1}{10}u + \frac{3}{10}v\right) = v$

$x + \frac{3}{10}u + \frac{9}{10}v = v$

$\boxed{x = -\frac{3}{10}u + \frac{1}{10}v}$

$T(\langle u, v\rangle)$

$\bar{r}(u,v) = \langle -\frac{3}{10}u + \frac{1}{10}v,\ \frac{1}{10}u + \frac{3}{10}v \rangle$

$\bar{r}_u = \langle -\frac{3}{10}, \frac{1}{10} \rangle$

$\bar{r}_v = \langle \frac{1}{10}, \frac{3}{10} \rangle$

$\|\bar{r}_u \times \bar{r}_v\|$

$\langle -\frac{3}{10}, \frac{1}{10}, 0 \rangle, -\frac{3}{10}, \frac{1}{10}$

$\langle \frac{1}{10}, \frac{3}{10}, 0 \rangle, \frac{1}{10}, \frac{3}{10}$

$\langle 0, 0, -\frac{9}{100} - \frac{1}{100} \rangle =$

$\langle 0, 0, -\frac{1}{10} \rangle$

$= \|\sqrt{0^2 + 0^2 + (-\frac{1}{10})^2}\| = \frac{1}{10}$

$$\begin{vmatrix} -\frac{3}{10} & \frac{1}{10} \\ \frac{1}{10} & \frac{3}{10} \end{vmatrix} = \left| -\frac{9}{100} - \frac{1}{100} \right| = \frac{1}{10}$$

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$