6^{th} Edition of Stewart.

This corresponds to your Chapter 15.

Looking at my exercise handouts, it looks like they align pretty well.

15.1 Double Integrals

15.2 Iterated Integrals

15.3 Double Integrals over General Regions

15.4 Polar Coordinates

15.5 Applications of Double Integrals

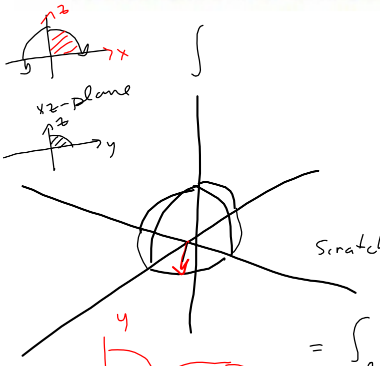
15.6 Triple Integrals

15.7 Cylindrical Coordinates

15.8 Spherical Coordinates

15.9 Change of Variables.

18. Evaluate $\iiint_E (x^3 + xy^2)\,dV$, where E is the solid in the first octant that lies beneath the paraboloid $z = 1 - x^2 - y^2$.



$z = 1 - (x^2+y^2) = 1 - r^2$

$$\int_0^1 \int_0^{\frac{\pi}{2}} \int_0^{1-r^2} (x^3+xy^2)\, dz\, r\, d\theta\, dr$$

$z=0 \implies x^2+y^2=1 \implies r^2=1$

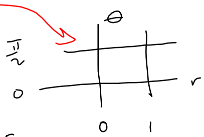
$r=1$ (QI)

Scratch: $x(x^2+y^2) = r\cos\theta\, r^2$

$$= \int_0^1 \int_0^{\frac{\pi}{2}} r^3\cos\theta\, r\, dz\, d\theta\, dr$$

$$= \int_0^1 \int_0^{\frac{\pi}{2}} \int_0^{1-r^2} r^4\cos\theta\, dz\, d\theta\, dr$$

$$= \int_0^1 \int_0^{\frac{\pi}{2}} \Big[[r^4\cos\theta] z \Big]_0^{1-r^2} d\theta\, dr$$



$$\int_0^1 \int_0^{\frac{Pi}{2}} \int_0^{1-r^2} r^4 \cdot \cos(t)\, dz\, dt\, dr = \frac{2}{35}$$

$$= \int_0^1 \int_0^{\frac{\pi}{2}} r^4\cos\theta\,[1-r^2]\, d\theta\, dr$$

$\sqrt{x^2} = |x|$

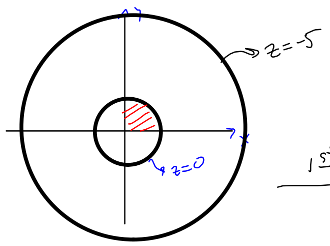
$$= \int_0^1 (r^4 - r^6)\,dr \int_0^{\frac{\pi}{2}} \cos\theta\, d\theta = \left[\frac{r^5}{5} - \frac{r^7}{7}\right]_0^1 \left[\sin\theta\right]_0^{\frac{\pi}{2}} = \left[\frac{1}{5}\cdot\frac{7}{7} - \frac{1}{7}\cdot\frac{5}{5}\right]\left[1-0\right]$$

What Theorem?

Fubini

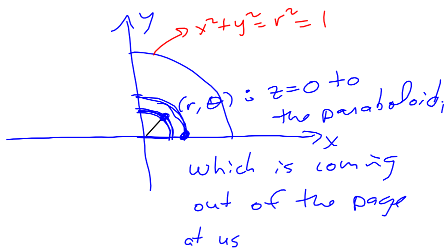
$= \frac{2}{35}$! Sweet!

$$\int_0^{\frac{\pi}{2}} \int_0^1 \int_0^{1-r^2} r^4\cos\theta\, dz\, dr\, d\theta = \frac{2}{35}, \text{ also}$$

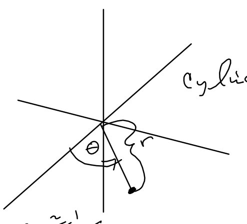


1st octant

1st Quadrant



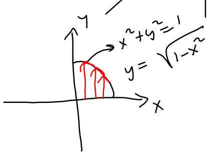
$$= \int_0^1 \int_0^{\frac{\pi}{2}} \int_0^{1-r^2} r^4\cos\theta\, dz\, d\theta\, dr$$



Cylindrical

Rectangular: Easy to formulate.

Hard to evaluate



$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{1-x^2-y^2} (x^3+xy^2)\, dz\, dy\, dx$$

§15.9 on the horizon.

" T-1 "