

14.5 #45

45. If $z = f(x, y)$, where $x = r \cos \theta$ and $y = r \sin \theta$, (a) find $\partial z / \partial r$ and $\partial z / \partial \theta$ and (b) show that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial z}{\partial \theta}\right)^2$$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dr} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dr} = f_x x_r + f_y y_r = f_x \cos \theta + f_y \sin \theta$$

$$f_\theta = f_x x_\theta + f_y y_\theta = -f_x r \sin \theta + f_y r \cos \theta$$

$$\Rightarrow f_r^2 + \frac{1}{r^2} f_\theta^2 = f_x^2 \cos^2 \theta + 2f_x f_y \sin \theta \cos \theta + f_y^2 \sin^2 \theta$$

$$+ \frac{1}{r^2} [f_x^2 r^2 \sin^2 \theta - 2f_x f_y r^2 \sin \theta \cos \theta + f_y^2 r^2 \cos^2 \theta]$$

$$= f_x^2 + f_y^2 \quad \square$$

38b $f(x,y)$ cts on $[a,b] \times [c,d]$

$$g(x,y) = \int_a^x \int_c^y f(s,t) dt ds \quad \forall (x,y) \in (a,b) \times (c,d)$$

$$g_{xy} = g_{yx} = f(x,y)$$

Fubini's theorem

$$\int_c^y f(s,t) dt$$

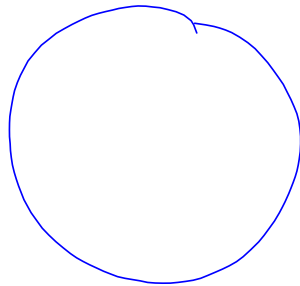
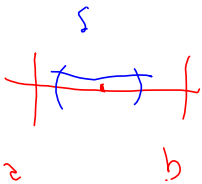
$$g_x = \int_c^y f(x,t) dt$$

$$g_{xy} = f(x,y) = g_{yx}$$

$$= \int_c^y \int_a^x f(s,t) ds dt$$

ϵ repeat

Completeness axiom.



$(a,b) \times (c,d)$

