14.5 #45

45. If $z = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, (a) find $\partial z/\partial r$ and $\partial z/\partial\theta$ and (b) show that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial r} = f_x x_r + f_y y_r = f_x \cos\theta + f_y \sin\theta$$

$$f_\theta = f_x x_\theta + f_y y_\theta = -f_x r\sin\theta + f_y r\cos\theta$$

$$\Rightarrow f_r^2 + \frac{1}{r^2} f_\theta^2 = f_x^2\cos^2\theta + 2f_xf_y\sin\theta\cos\theta + f_y^2\sin^2\theta$$

$$+ \frac{1}{r^2}\left[f_x^2 \cancel{r^2}\sin^2\theta - 2f_xf_y \cancel{r^2}\sin\theta\cos\theta + f_y^2\cancel{r^2}\cos^2\theta\right]$$

$$= f_x^2 + f_y^2 \quad \blacksquare$$

38b $f(x,y)$ cts on $[a,b]\times[c,d]$

$$g(x,y) = \int_a^x \int_c^y f(s,t)\,dt\,ds \quad \forall\ (x,y) \in (a,b)\times(c,d)$$

Fubini set

$$= \int_c^y \int_a^x f(s,t)\,ds\,dt$$

$$g_{xy} = g_{yx} = f(x,y)$$

$$\int_c^y f(s,t)\,dt$$

Σ repeat

$$g_x = \int_c^y f(x,t)\,dt$$

$$g_{xy} = f(x,y) = g_{yx}$$

Completeness axiom.

$(a,b)\times(c,d)$