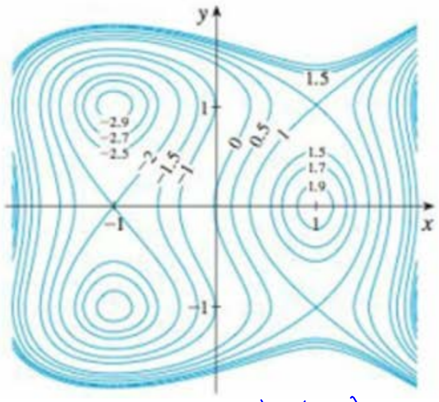
3–4 Use the level curves in the figure to predict the location of the critical points of f and whether f has a saddle point or a local maximum or minimum at each critical point. Explain your

4. $f(x, y) = 3x - x^3 - 2y^2 + y^4$



$f_x = 3 - 3x^2$

$f_y = -4y + 4y^3$

$f_{xx} = -6x$

$f_{yy} = -4 + 12y^2$

$f_{yx} = f_{xy} = 0$

$D = f_{xx}f_{yy} - (f_{xy})^2$

$= (-6x)(-4+12y^2) - 0$

$= (6x)(4-12y^2)$

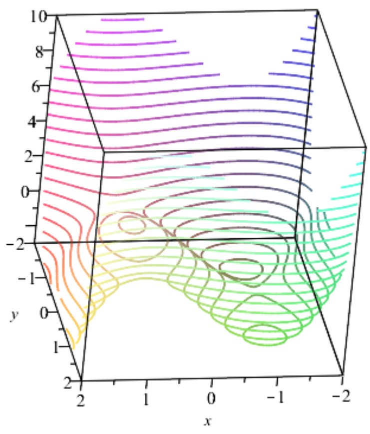
Critical: $(-1,1), (-1,0), (-1,-1)$
$(1,1), (1,0), (1,-1)$

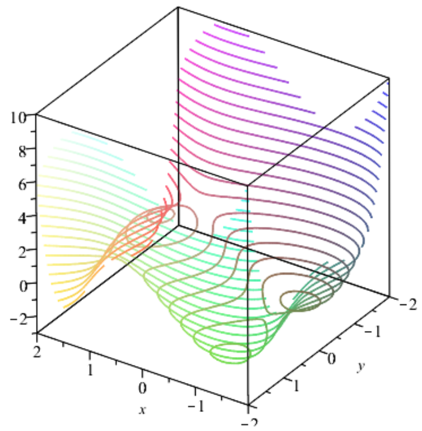
$D(-1,1) = 6(-1)(4-12(1)^2)$
$= -6(-8) = 48 > 0 \Rightarrow$

$\boxed{f_{xx}(-1,1) = -6(-1) = 6 > 0 \quad \text{Local min}}$

$D(\ldots) \ldots (-4+12(\ldots)^2)$

0



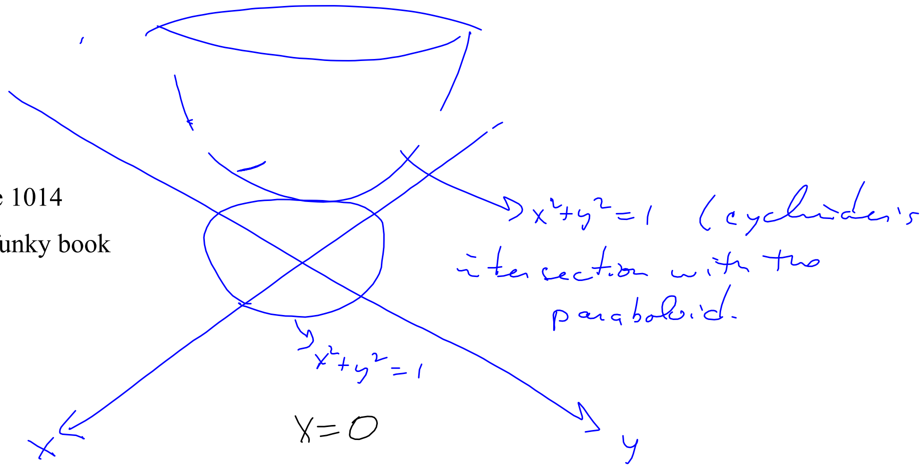


§14.8

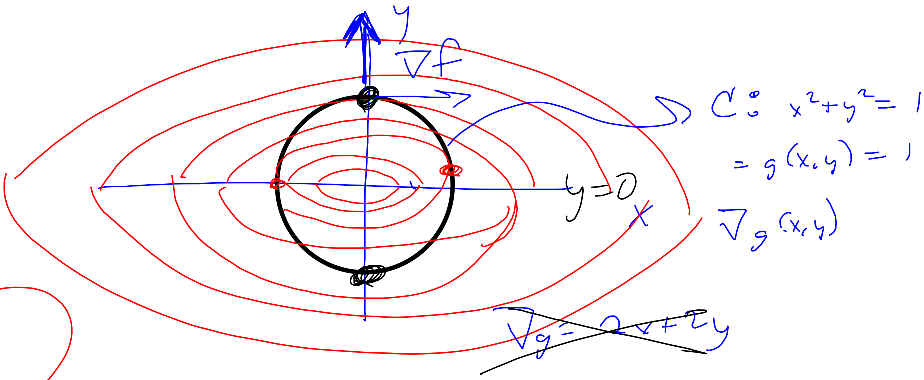
$g(x,y) = x^2+y^2 = 1$

$f(x,y) = x^2+2y^2$

See Page 1014
in your funky book



X=0



$x=0 \Rightarrow y=\pm 1$

$y=0 \Rightarrow x=\pm 1$

$\nabla g = \langle g_x, g_y \rangle = \langle 0, 2 \rangle$ @ $(0,1)$ $\qquad \nabla g = \langle 2x, 2y \rangle$

∇f is parallel to ∇g!!!

$\Rightarrow \nabla f = \lambda \nabla g$ for some λ!

Find points where this is satisfied

$\nabla f = \langle 2x, 4y \rangle = \lambda \langle 2x, 2y \rangle$

There're a lot of tricks, here.

$2x = 2\lambda x \quad , \quad 4y = 2\lambda y$

$\lambda = 1$ does it

$4y = 2\lambda y$

$2xy = 2\lambda xy \qquad 4xy = 2\lambda xy$

$4xy = 4\lambda xy \equiv 2\lambda xy$

$\lambda = 0$ OR $x = 0$ OR $y = 0$