§14.6 #34

$z = 1000 - .005x^2 - .01y^2$

$= 1000 - \frac{x^2}{200} - \frac{y^2}{100}$

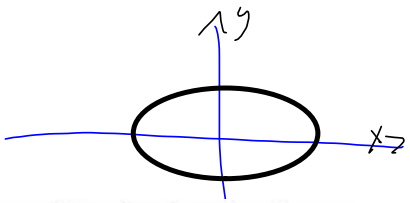
$z = 999$ trace:

$999 = 1000 - \frac{x^2}{200} - \frac{y^2}{100}$

$\sqrt{200} = \sqrt{2 \cdot 100} = 10\sqrt{2}$

$\sqrt{100} = 10$

$\Rightarrow \frac{x^2}{200} + \frac{y^2}{100} = 1$



34. Suppose you are climbing a hill whose shape is given by the equation $z = 1000 - 0.005x^2 - 0.01y^2$, where x, y, and z are measured in meters, and you are standing at a point with coordinates (60, 40, 966). The positive x-axis points east and the positive y-axis points north.

(a) If you walk due south, will you start to ascend or descend? At what rate?

(b) If you walk northwest, will you start to ascend or descend? At what rate?

(c) In which direction is the slope largest? What is the rate of ascent in that direction? At what angle above the horizontal does the path in that direction begin?

$z = f(x,y) = 1000 - .005x^2 - .01y^2$

$f_x = -.01x$

$f_y = -.02y$

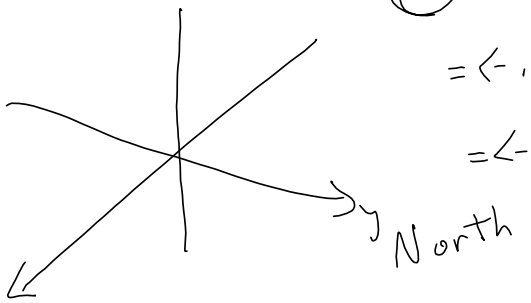
$\nabla f = \langle -.01x, -.02y \rangle$

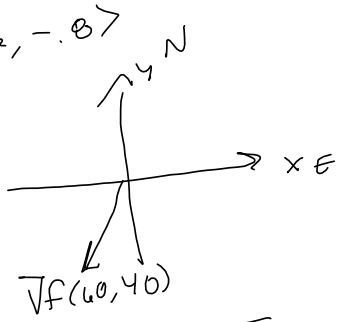
$(60, 40, f(60,40))$

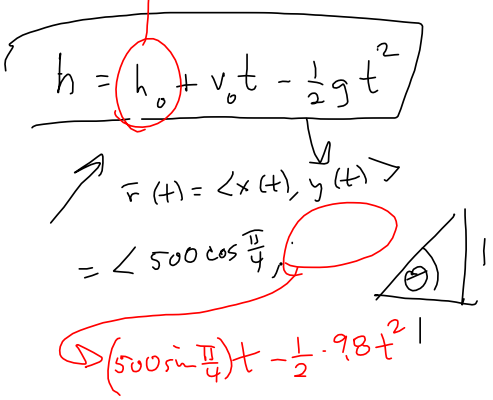
(c) $\nabla f(60,40)$

$= \langle -.01(60), -.02(40) \rangle$

$= \langle -.6, -.8 \rangle$







Rate of ascent is $\|\nabla f(60,40)\| = \sqrt{.6^2 + .8^2}$

= steepness as $= \sqrt{.36 + .64}$

$= 1$

$\theta = \frac{\pi}{4} = 45^\circ$

§14.7 Max & Min. Nostalgia Tour of Calc I

If $f(x,y)$ is smooth and (x_0,y_0,z_0) is a max/min, then

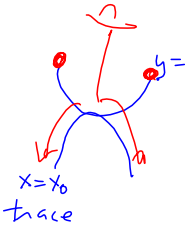
$$f_x(x_0,y_0) = 0$$

$$f_y(x_0,y_0) = 0$$

---

f smooth, (x_0,y_0,z_0) an extrem $\Rightarrow$ $f_x(x_0,y_0) = f_y(x_0,y_0) = 0$

"$\Leftarrow$" doesn't hold.



A saddle is a problem, here.

$$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx}f_{yy} - f_{xy}f_{yx}$$

$$= f_{xx}f_{yy} - (f_{xy})^2 = D$$

$= f_{xy}$! Clairaut

$D > 0$ (i) $f_{xx} > 0 \Rightarrow$ min

(ii) $f_{xx} < 0 \Rightarrow$ max

$D < 0$ Neither

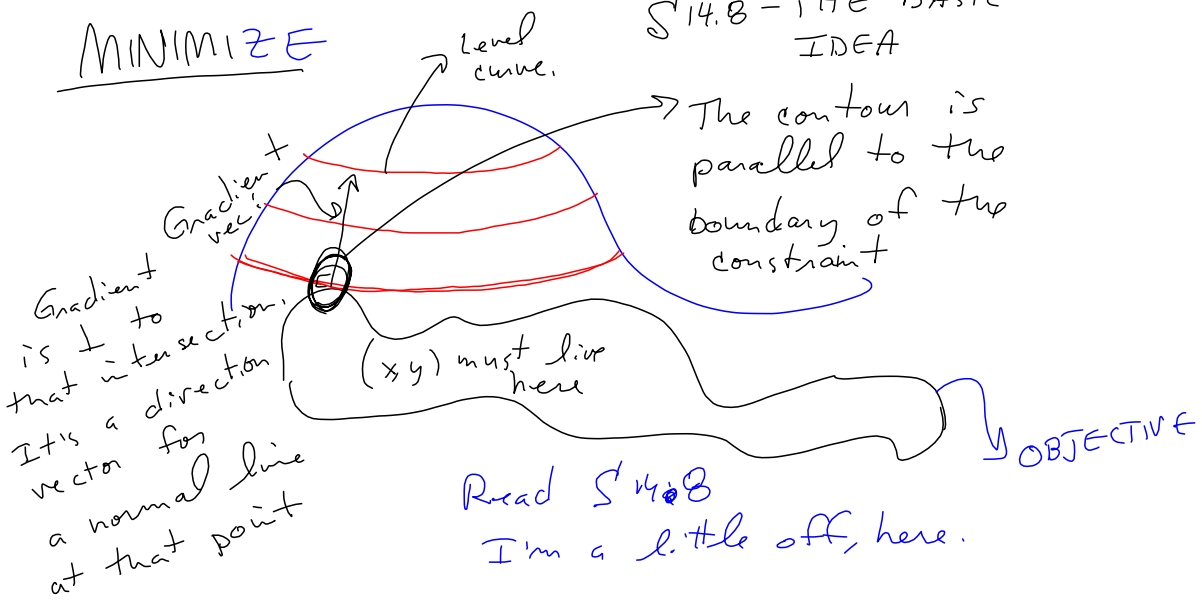
$D = 0$ My Dunno.

cheat sheet material

---

§14.8 – THE BASIC IDEA

MINIMIZE



The contour is parallel to the boundary of the constraint

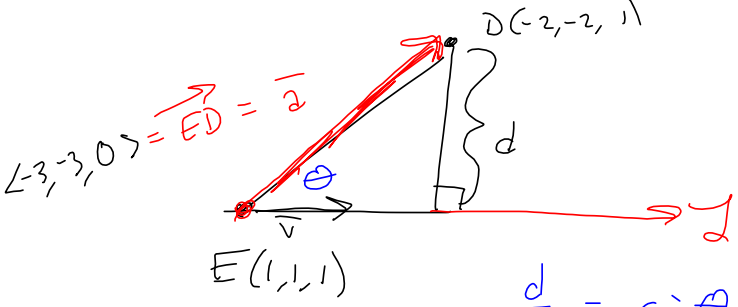
Read §14.8

I'm a little off, here.

Segment From
$A(1,2,3)$ to $B(5,-6,1)$

is $(1-t)\langle 1,2,3\rangle + t\langle 5,-6,1\rangle \quad 0 \le t \le 1$

Point to a line:



y: $\bar{r}(t) = \langle 1,1,1\rangle + t\langle 1,-2,3\rangle$

$= \bar{r}_0 + t\bar{v}$

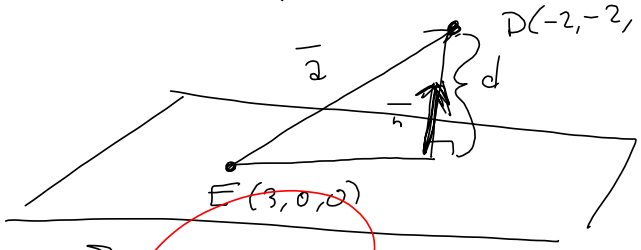
$$\frac{d}{\|\bar{a}\|} = \sin\theta = \frac{\|\bar{a}\times\bar{v}\|}{\|\bar{a}\|\|\bar{v}\|}$$

$$d = \frac{\|\bar{a}\times\bar{v}\|}{\|\bar{v}\|}$$

Point to a plane:

P: $2x-3y+5z=6$

$\langle 2,-3,5\rangle = \bar{n}$



$\overrightarrow{ED} = \langle -5,-2,1\rangle$

$$d = \left\|\text{proj}_{\bar{n}}\bar{a}\right\| = \left|\left(\frac{\bar{a}\cdot\bar{n}}{\|\bar{n}\|}\right)\left(\frac{\bar{n}}{\|\bar{n}\|}\right)\right|$$

$$= \frac{|-10+6+5|}{\sqrt{4+9+25}} = \frac{1}{\sqrt{38}}$$