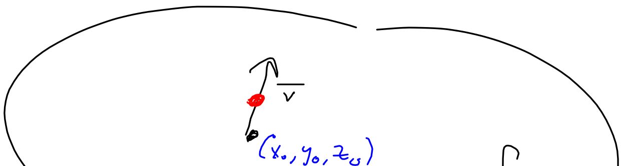
Questions?

§14.6 Directional Derivatives

There's video up, but it's a little clumsy



on surface.

we find $D_{\bar{v}} f(x,y)$ = Directional Derivative in direction of $\bar{v}$ @ (x_0, y_0, z_0)

$$\frac{f(x_0+ha, y_0+hb) - f(x_0, y_0)}{h}$$

$z = f(x,y)$ thought of as a vector function

$\bar{F}(x,y) = \bar{r}$

Then $\boxed{f(x_0+ha, y_0+hb) = f(\bar{r}_0 + h\bar{v})}$

$\bar{r}_0 = \langle x_0, y_0, f(x_0, y_0)\rangle$

Viewing as vector inputs, then

$f(\langle x_0+ha, y_0+hb\rangle) = f(\bar{r}_0 + h\bar{v})$

is written the same, no matter how many dimensions.

$f(x_0+ha, y_0+hb, z_0+hc)$

$$f\underbrace{(x_0+ha, y_0+hb, z_0+hc, w_0+hd, k_0+he\rangle}_{}$$

$\bar{r}_0 + h\bar{v}$ doesn't matter how many variables, the symbolic vector notation is identical.

Directional Derivative:

$$\lim_{h\to 0} \frac{f(x+ha, f(y+hb)) - f(x,y)}{h} = D_{\bar{u}} f(x,y)$$

Video does a decent job explaining this formula:

If $\bar{u} = \langle a, b\rangle$ is a unit vector.

Then $D_{\bar{u}} f(x,y) = f_x(x,y)\, a + f_y(x,y)\, b$

We can write this very nicely!

Video explains why, but it basically comes down to the x-direction & the y-direction "span" the plane.

$\bar{i} = \langle 1, 0\rangle$ & $\langle 0, 1\rangle$

$$\boxed{\bar{u} = \langle a, b\rangle = a\langle 1, 0\rangle + b\langle 0, 1\rangle}$$

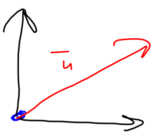
f_x gives change in x-direction

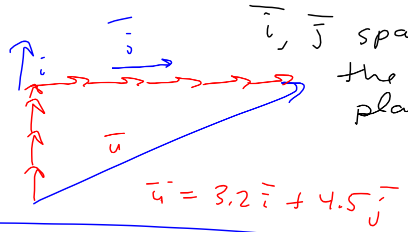
f_y " " " "

It makes sense that $D_{\bar{u}} f$

= a · change in x-direction

+ b · " " y-direction





$\bar{i}, \bar{j}$ span the plane.

$$\boxed{D_{\bar{u}} f(x,y) = f_x(x,y)\, a + f_y(x,y)\, b}$$

$= \langle f_x(x,y), f_y(x,y)\rangle \cdot \langle a, b\rangle$

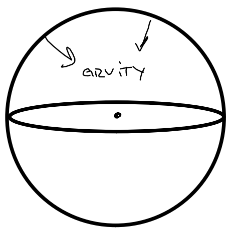
$= \nabla f \cdot \bar{u}$

$\|\bar{u}\| = 1$ is only way this works.

∂, ∇ — Del

Method $f_x, f_y, \dfrac{\bar{u}}{\|\bar{u}\|}$

$\langle f_x, f_y\rangle \cdot \dfrac{1}{\|\bar{u}\|}\bar{u}$

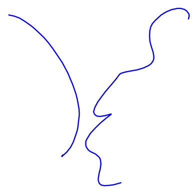


Force of Gravity
Intensity of light
Electrostatic Field.
we can assign a vector to each point giving magnitude & direction of force.

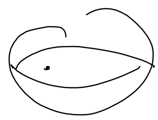
$F = 10$ at 20 ft
$= \frac{10}{2^2}$ at double 20 ft

$$F = G\frac{m_1 m_2}{r^2}$$

Sound intensity
Light intensity
proportional to the reciprocal square of the distance.
of the distance



Think about level surfaces for a function in 3-space

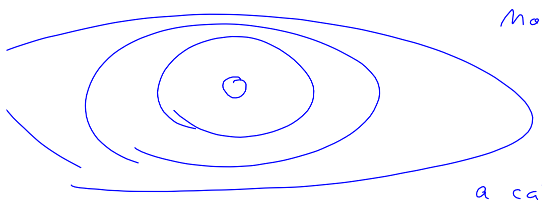


Really hard to "see" a function of 3 variables in space.

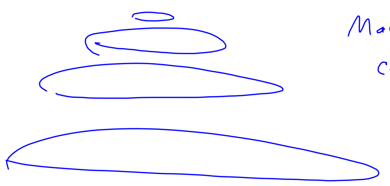
$F(x,y,z) =$ Konstant is a level SURFACE

A sphere about a point charge is a level surface corresponding to charge. The charge propagates in all directions at the same speed. Sometimes this is as close as we can come to visualizing 4-D stuff.

Level curves for surfaces?



Mountain
2-D surface embedded in 3-space.
Contours took a cat scan of the surface



Mountain via contours.

Now we have 3-D objects embedded in 4-space. Level surfaces are the analogy.

Consider $\underbrace{z = f(x,y)}_{\text{Surface}} \Rightarrow \underbrace{f(x,y) - z = 0}_{\text{Level surface for}}$

$F(x,y,z) = f(x,y) - z$

$F(x,y,z) = K$

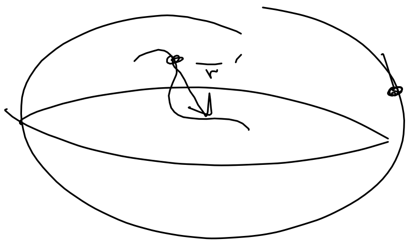
$F(x(t), y(t), z(t)) = K$

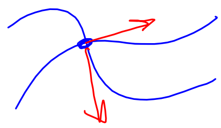
t is THE independent variable

$$\frac{d}{dt} F = F_x x_t + F_y y_t + F_z z_t$$

$$= \langle F_x, F_y, F_z \rangle \cdot \langle x'(t), y'(t), z'(t) \rangle = 0$$

Let $\bar{r} = \langle x(t), y(t), z(t) \rangle$ be any curve on surface. Then $\bar{r}'(t)$ lies "in" the tangent plane.



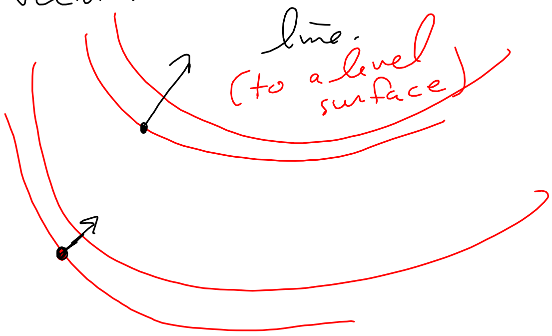


$\nabla F \cdot \bar{r}' = 0$ says $\nabla f \perp$ to tangent plane.

$\nabla F = \bar{n}$ = normal vector to tangent plane to the level surface

Also ∇f is a Direction vector for the normal line. (to a level surface)

Gradient points uphill.



Maximize $D_{\bar{u}}$ at a point.

$D_{\bar{u}} = f_x a + f_y b$

What $\bar{u}$ makes $D_{\bar{u}}$ biggest?

Want this maximal

$$D_{\bar{u}} = \langle f_x, f_y \rangle \cdot \langle a, b \rangle$$

$$= \nabla f \cdot \bar{u} = \|\nabla f\| \|\bar{u}\| \cos\theta$$

$$\leq \|\nabla f\| \|\bar{u}\|$$

$$\cos\theta = 1 \ @ \ \theta = 0$$

$$\Longrightarrow \nabla f \parallel \bar{u}$$

i.e. ∇f is the direction of greatest increase in surface.

∇f always points uphill.