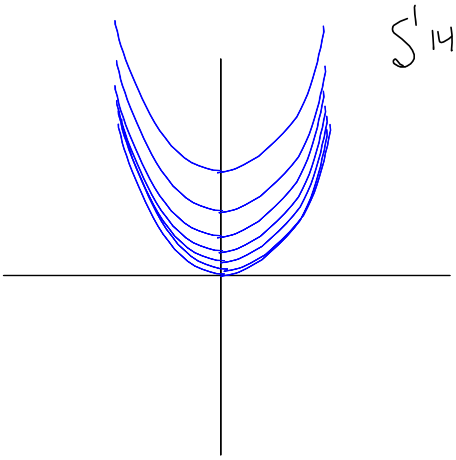
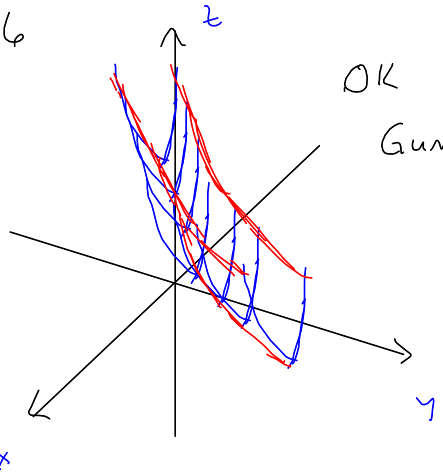


§14.1 #36



OK 4
GUMMINT
WORK.

The plan is to get through theory in class and in video, and free up some face-to-face time for Computer Algebra.

Section 14.4 Tangent Planes and Linear Approximations

We know from Equation 12.5.7 that any plane passing through the point $P(x_0, y_0, z_0)$ has an equation of the form

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

$$\Delta y = f(x+\Delta x) - f(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x}\Delta x \approx f'(x)\Delta x \approx \Delta y$$

$\Delta y \approx dy$, $\Delta x \equiv dx$

$$\boxed{1} \quad z - z_0 = a(x - x_0) + b(y - y_0)$$

$z = a(x-x_0) + b(y-y_0) + z_0$

Tangent LINE @ (x_0, y_0, z_0)

in the x-direction:

$z = z_0 + \frac{\partial f}{\partial x}(x-x_0)$ y fixed @ $y = y_0$

in the y-direction:

$z = z_0 + \frac{\partial f}{\partial y}(y-y_0)$ $x = x_0$ fixed.

$z = \frac{\partial f}{\partial y}(y-y_0) + z_0$

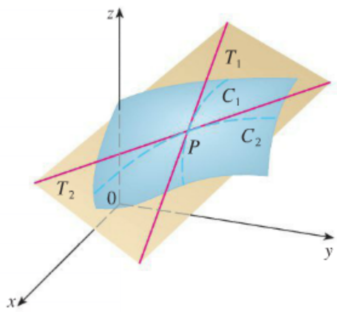


FIGURE 1
The tangent plane contains the tangent lines T_1 and T_2.

$\boxed{2}$ Suppose f has continuous partial derivatives. An equation of the tangent plane to the surface $z = f(x, y)$ at the point $P(x_0, y_0, z_0)$ is

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$z = f_x(x_0, y_0)(x-x_0) + f_y(x_0, y_0)(y-y_0) + z_0$

$\cancel{y - y_0 = m(x-x_0)}$

$y - y_0 = m(x-x_0)$

$y = m(x-x_0) + y_0$

Recall tangent lines in the plane:

Smooth curves and surfaces are locally linear.

TEC Visual 14.4 shows an animation of Figures 2 and 3.

ing the domain of the function $f(x, y) = 2x^2 + y^2$. Notice that the more we zoom in, the flatter the graph appears and the more it resembles its tangent plane.

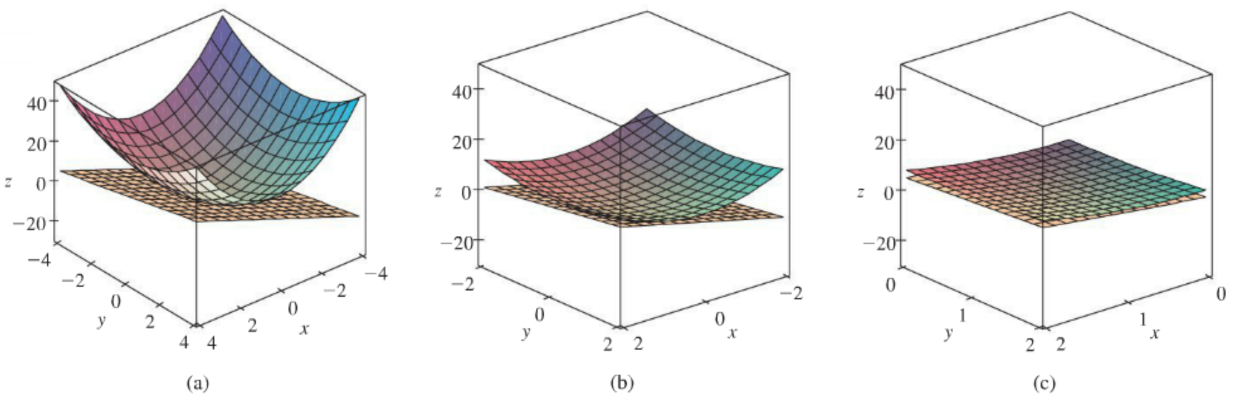


FIGURE 2 The elliptic paraboloid $z = 2x^2 + y^2$ appears to coincide with its tangent plane as we zoom in toward (1, 1, 3).

Tangent Planes in 3-space also make for nice approximations:

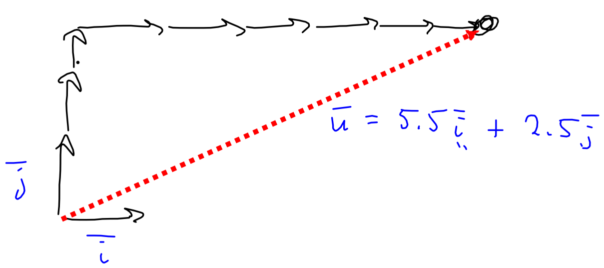
For this example: $L_{(1,1)}(x,y) = f_x(1,1)(x-1) + f_y(1,1)(y-1) + 3$

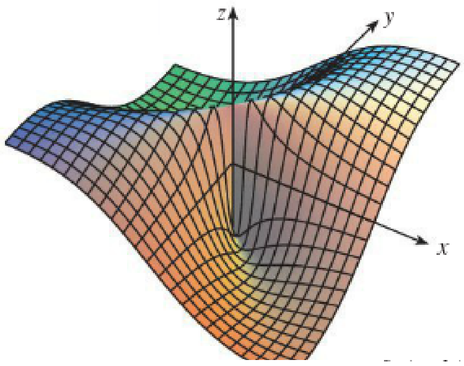
$f_x = 4x \rightsquigarrow 4(1) = 4$

$f_y = 2y \rightsquigarrow 2(1) = 2$

$z = 4(x-1) + 2(y-1) + 3$

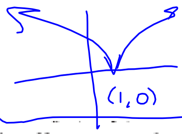
Good approximation for the elliptic paraboloid in a small neighborhood of (1,1,3).





This one has a cusp at the origin, its derivatives of all orders exist, but they aren't continuous at the origin.

2-D: $f(x) = (x-1)^{2/3}$ @ $x = 1$



Not @ (0,0), I don't think.

So a function of two variables can behave badly even though both of its partial derivatives exist.

FIGURE 4

$f(x, y) = \dfrac{xy}{x^2 + y}$ if $(x, y) \neq (0, 0)$,

$f(0, 0) = 0$

$\Delta x \equiv dx$

Increment of y:

5 $\quad \Delta y = f'(a)\,\Delta x + \varepsilon\,\Delta x \quad$ where $\varepsilon \to 0$ as $\Delta x \to 0$

$\Delta y \approx f'(a)\,dx = f'(a)\,\Delta x \qquad dy = f'(x)\,dx$

Increment of z:

6 $$\Delta z = f(a + \Delta x, b + \Delta y) - f(a, b)$$

7 Definition If $z = f(x, y)$, then f is **differentiable** at (a, b) if Δz can be expressed in the form

$$\boxed{\Delta z = f_x(a, b)\,\Delta x + f_y(a, b)\,\Delta y + \varepsilon_1\,\Delta x + \varepsilon_2\,\Delta y}$$

where ε_1 and $\varepsilon_2 \to 0$ as $(\Delta x, \Delta y) \to (0, 0)$. $\quad \Delta z \approx f_x(a,b)\Delta x + f_y(a,b)\Delta y$

If you want to play with these ideas (and formalisms), the #46 is the bomb.

If you *don't*, then the following is a very practical way to check for differentiability is given by:

8 Theorem If the partial derivatives f_x and f_y exist near (a, b) and are continuous at (a, b), then f is differentiable at (a, b).

Everything's nice & smooth on its domain, generally.

$\log_b(0 \text{ or less})$ $\quad \dfrac{\text{~~~}}{0}$ $\quad$ & $\sqrt{\text{Negative}}$ are bad.

$\log_5(x)$ Need $x > 0$

$\sqrt[2n]{x}$ Need $x \geq 0$

$\dfrac{\text{Stuff}}{x}$ Need $x \neq 0$

Differentials in the Plane:

9 $$dy = f'(x)\,dx$$

The Differential of a surface in 3-space:

10 $$\boxed{dz = f_x(x, y)\,dx + f_y(x, y)\,dy = \frac{\partial z}{\partial x}\,dx + \frac{\partial z}{\partial y}\,dy}$$

Also called the "total differential."

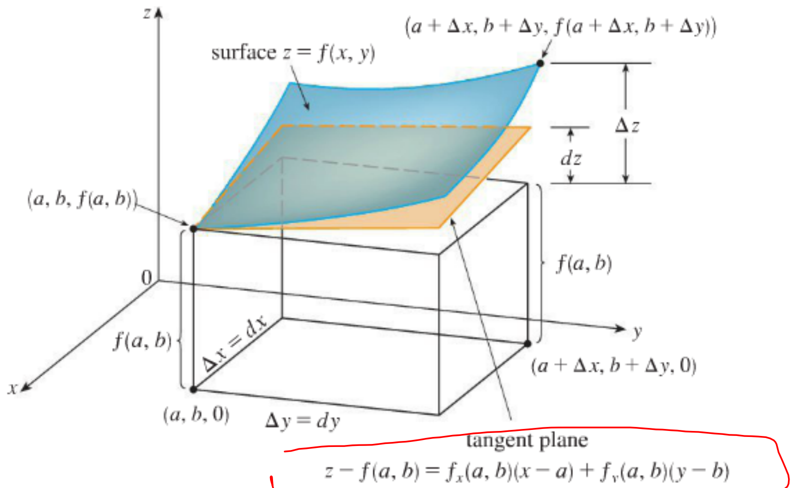


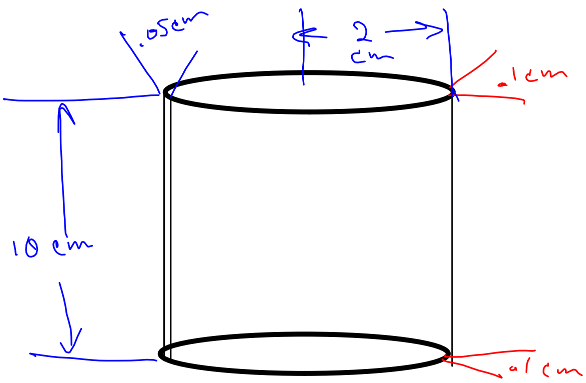
FIGURE 7

is eq'n of tan plane.

$$f(x, y) \approx z = f_x\,(x-a) + f_y\,(y-b) + f(a, b)$$

$$y_{(a,b)}^{(x,y)}$$

36. Use differentials to estimate the amount of metal in a closed cylindrical can that is 10 cm high and 4 cm in diameter if the metal in the top and bottom is 0.1 cm thick and the metal in the sides is 0.05 cm thick.



We use volume of the can and use the thicknesses to estimate the volume of metal as a change in volume of the can.

r = radius (cm)

h = height (cm)

$z = V = \pi r^2 h$

$V_r = 2\pi r h \rightsquigarrow 2\pi(2)(10) = 40\pi$

$dV = V_r \Delta r + V_h \Delta h$

$V_h = \pi r^2 \rightsquigarrow \pi(2)^2 = 4\pi$

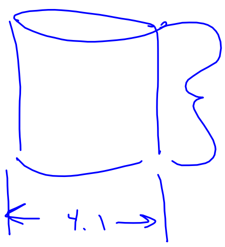
$dV = 40\pi \cdot .05 + 4\pi \cdot .2$

$= \frac{14\pi}{5} \approx 8.796459431$

$\Delta r = .05$ cm

$\Delta h = 2 \cdot 0.1 = 0.2$ cm

TOP AND BOTTOM



$$g(x,y) = 6 - x - x^2 - 2y^2$$

$$L_{(1,2)}(x,y)$$