Shorthand:

There is, there exists $\exists$ such that or so that $\ni$

For all, for each, for every $\forall$

x is in the set S, x is an element of S $x \in S$

A implies B $A \Rightarrow B$

A implies B and B implies A, if and only if, A is necessary and sufficient to B

$A \Leftrightarrow B$ or A iff B

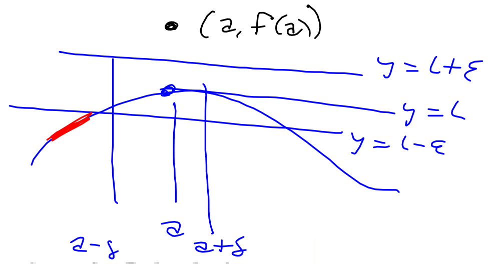
Recall the definition of limit from Calculus I:

$\lim_{x \to a} f(x) = L$ means:

$\forall\, \varepsilon > 0\ \exists\, \delta > 0 \ni 0 < |x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon.$

"$f(x)$ is continuous" means

$\lim_{x \to a} f(x) = f(a)$



1 Definition Let f be a function of two variables whose domain D includes points arbitrarily close to (a, b). Then we say that the **limit of $f(x, y)$ as (x, y) approaches (a, b)** is L and we write

$$\lim_{(x,y)\to(a,b)} f(x, y) = L$$

if for every number $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that

if $(x, y) \in D$ and $0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta$ then $|f(x, y) - L| < \varepsilon$

Trick: Fix x & let $y \to b$

or Fix y & let $x \to a$

§14.2 Look for ways it might NOT work.

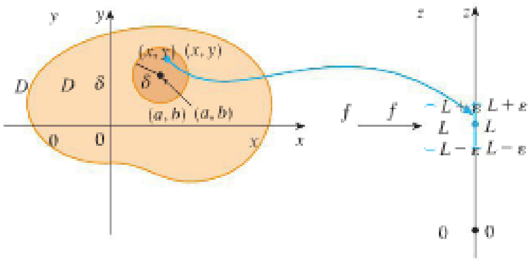


FIGURE 1

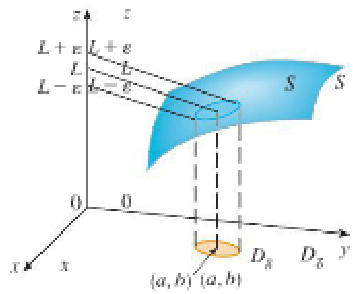


FIGURE 2

If $f(x, y) \to L_1$ as $(x, y) \to (a, b)$ along a path C_1 and $f(x, y) \to L_2$ as $(x, y) \to (a, b)$ along a path C_2, where $L_1 \neq L_2$, then $\lim_{(x,y)\to(a,b)} f(x, y)$ does not exist.

When the limit does NOT exist, sometimes the only practical way to show it is to be clever in how you make the approach to the limiting input value from a direction (or along a curve) where you get two different results, proving the limit doesn't exist.

Standard "trick."

EXAMPLE 1 Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$ does not exist.

? a rotating line on
e 6 shows differ-
gin from different

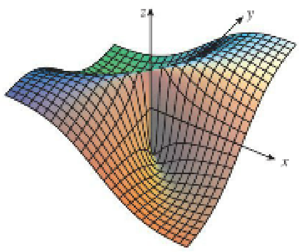


FIGURE 6
$f(x, y) = \frac{xy}{x^2 + y^2}$

Maybe y'all can help me grok the enrichment tool:

https://www.cengage.com/math/discipline_content/stewartcalc7/2008/14_cengage_tec/publish/deployments/transcendentals_7e/7e_m12_6a.html

EXAMPLE 4 Find $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2}$ if it exists.

It DOES!!!