

14.1 #s 13, 15, 17-19, 23, 24, 36, 41, 43, 45, 53, 54, 57, 58

Definition A function f of two variables is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is, $\{f(x, y) \mid (x, y) \in D\}$.

Definition If f is a function of two variables with domain D , then the **graph** of f is the set of all points (x, y, z) in \mathbb{R}^3 such that $z = f(x, y)$ and (x, y) is in D .

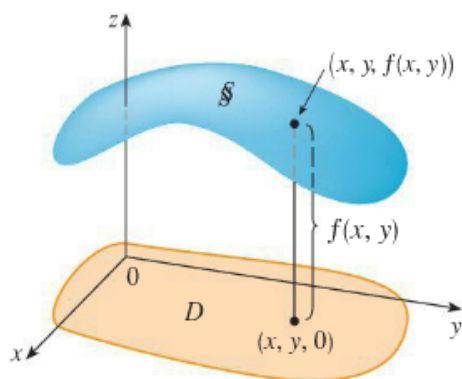


FIGURE 5

EXAMPLE 5 Sketch the graph of the function $f(x, y) = 6 - 3x - 2y$.

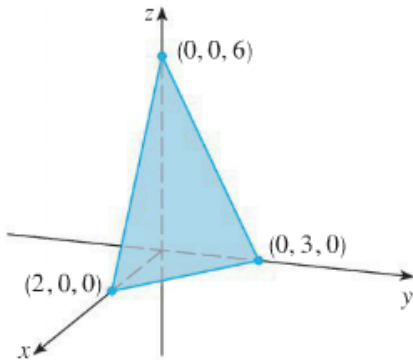


FIGURE 6

EXAMPLE 6 Sketch the graph of $g(x, y) = \sqrt{9 - x^2 - y^2}$. *Top half of sphere*

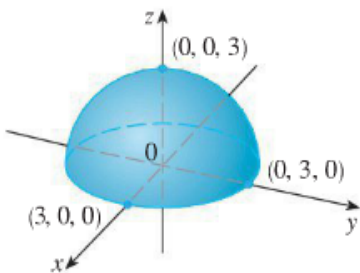


FIGURE 7
Graph of $g(x, y) = \sqrt{9 - x^2 - y^2}$

$$z = \sqrt{9 - x^2 - y^2}$$

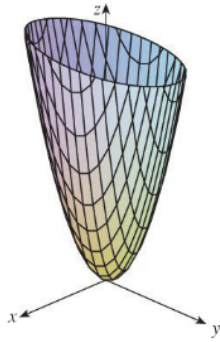
$$z^2 = 9 - x^2 - y^2$$

$$x^2 + y^2 + z^2 = 9$$

$$z^2 = 9 - x^2 - y^2$$

$$z = \pm \sqrt{9 - x^2 - y^2}$$

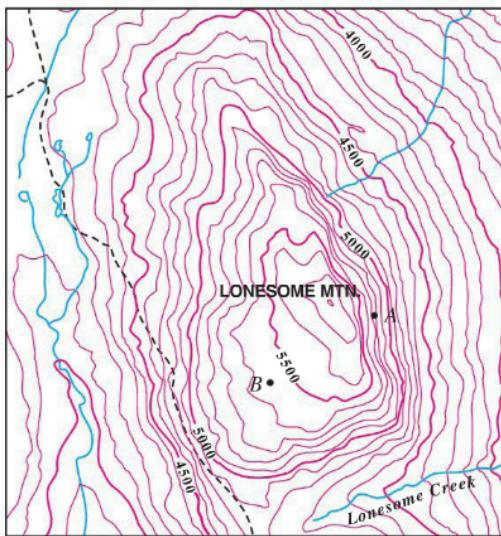
EXAMPLE 8 Find the domain and range and sketch the graph of $h(x, y) = 4x^2 + y^2$.



$z=1: \quad \frac{x^2}{\frac{1}{4}} + \frac{y^2}{1} = 1$

 $z=10: \text{ Same shape. BIGGER.}$

Definition The level curves of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant (in the range of f).



Topographic Maps!

FIGURE 12

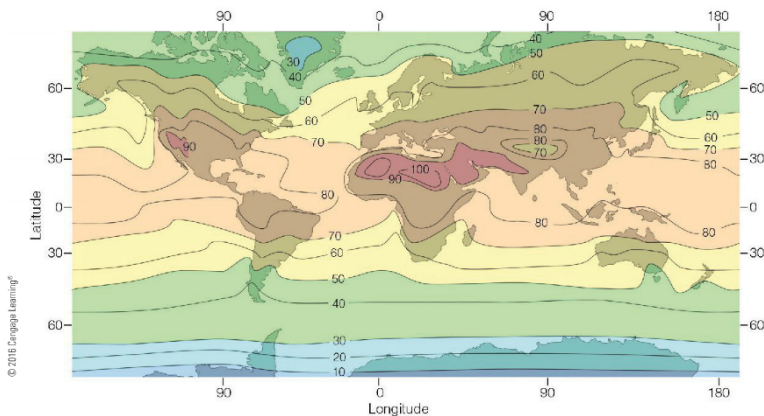
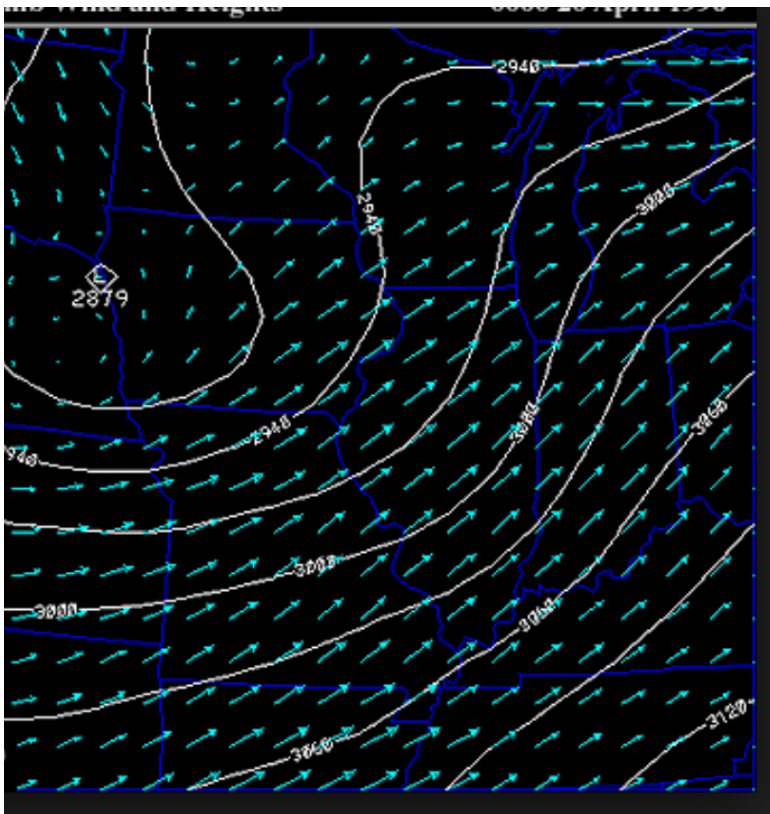


FIGURE 13 Average air temperature near sea level in July ($^{\circ}\text{F}$)



EXAMPLE 12 Sketch some level curves of the function $h(x, y) = 4x^2 + y^2 + 1$

This is our friend, the elliptical paraboloid, opening up, with vertex at $(0, 0, 1)$.

11-20 Find and sketch the domain of the function.

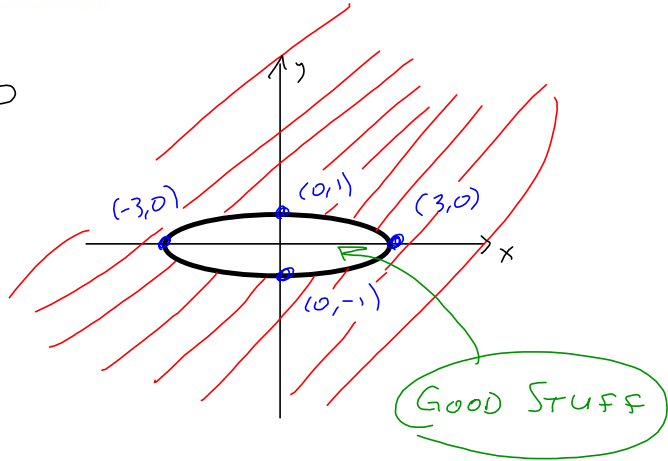
13. $f(x, y) = \ln(9 - x^2 - 9y^2)$

Scratch out the bad stuff. Need $9 - x^2 - 9y^2 > 0$

$$x^2 + 9y^2 < 9$$

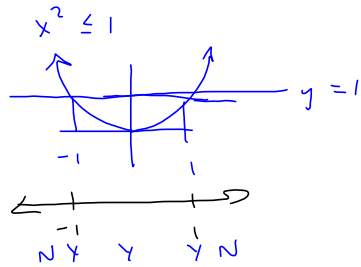
$$\frac{x^2}{9} + y^2 < 1$$

$$\frac{x^2}{3^2} + \frac{y^2}{1^2}$$



15. $f(x, y) = \sqrt{1 - x^2} - \sqrt{1 - y^2}$

Need $1 - x^2 \geq 0$ and $1 - y^2 \geq 0$



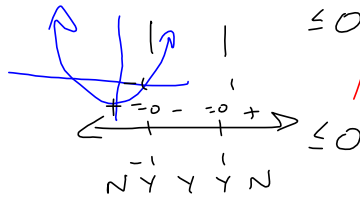
$$1 - x^2 \geq 0$$

$$(1-x)(1+x) \geq 0$$

$$-(x-1)(x+1) \geq 0$$

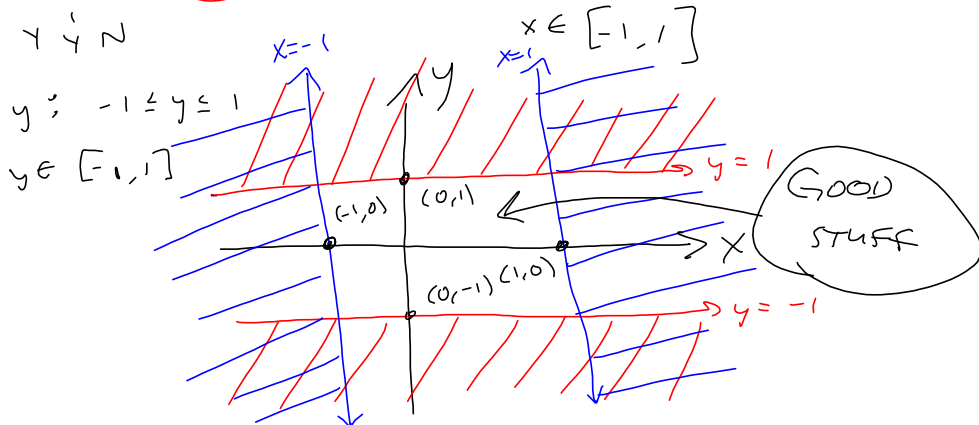
$$(x-1)(x+1) \leq 0$$

Test $x = 2$
 $(2-1)(2+1) > 0$ (+)



AND
 $1 - y^2 \geq 0 \Rightarrow$
 $y^2 - 1 \leq 0$
 $(y-1)(y+1) \leq 0$
 $+ \quad =0 \quad - \quad =0 \quad +$
 ≤ 0
 $N \quad Y \quad Y \quad N$

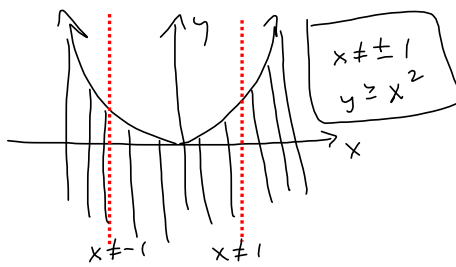
Find one sign.
 Manage sign changes.



$$17. f(x, y) = \frac{\sqrt{y - x^2}}{1 - x^2}$$

Scratch out the bad stuff. Keep the good stuff.

Need $y - x^2 \geq 0$ and $1 - x^2 \neq 0$
 $y \geq x^2$



$$18. f(x, y) = \arcsin(x^2 + y^2 - 2)$$

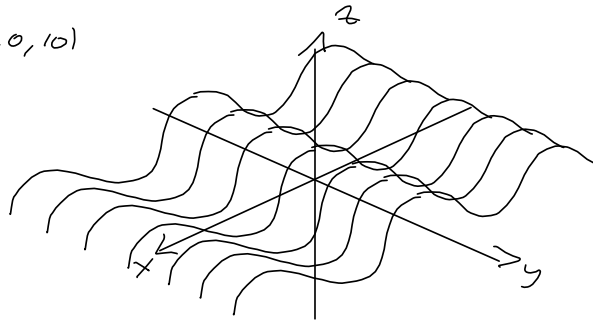
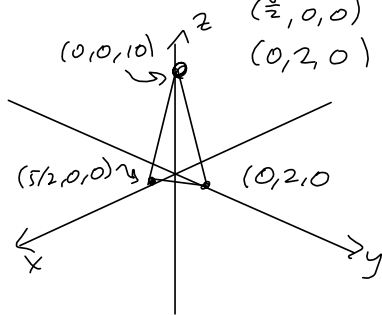
19. $f(x, y, z) = \sqrt{1 - x^2 - y^2 - z^2}$

21-29 Sketch the graph of the function.

23. $f(x, y) = 10 - 4x - 5y$

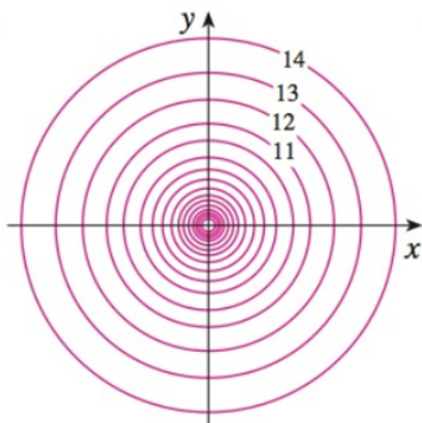
24. $f(x, y) = \cos x \quad z = \cos x$

$z = 10 - 4x - 5y \Rightarrow 4x + 5y + z = 10$
 $(\frac{5}{2}, 0, 0) \quad (0, 0, 10)$

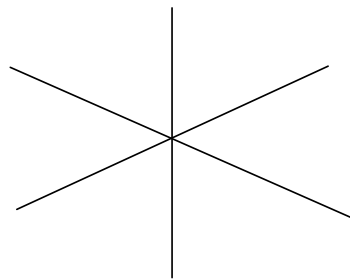
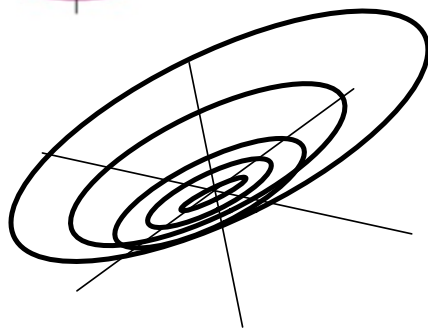
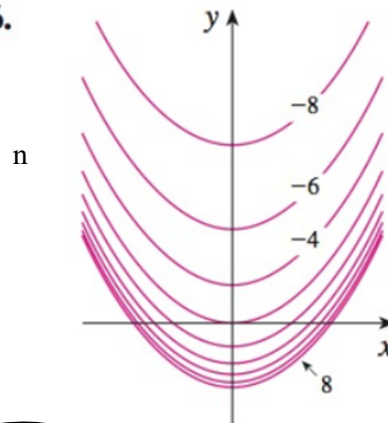


35-38 A contour map of a function is shown. Use it to make a rough sketch of the graph of f . #35 NA

35.



36.

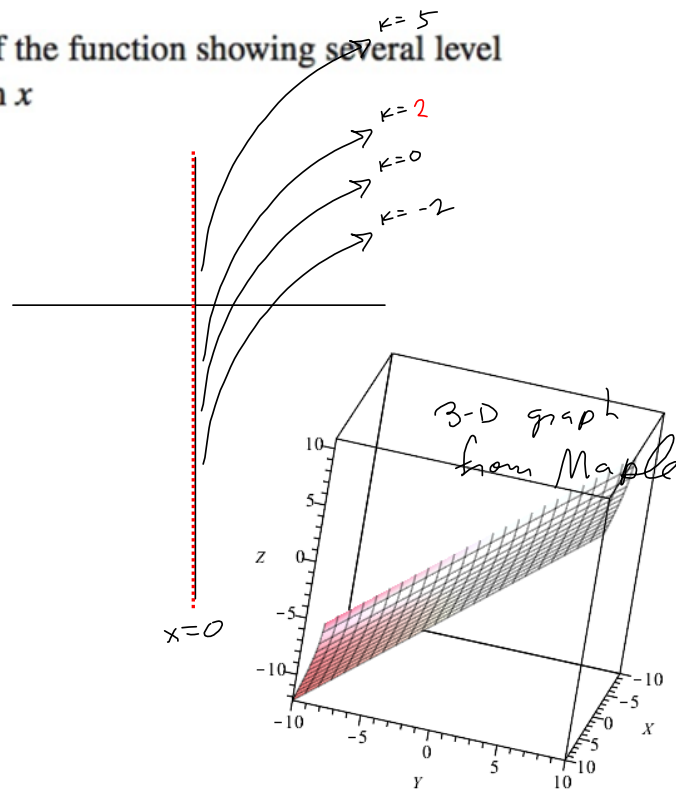


39-46 Draw a contour map of the function showing several level curves. **41.** $f(x, y) = y - \ln x$

$$y - \ln(x) = k$$

$$y = \ln(x) + k$$

$$y = \ln(x) + 2$$



43. $f(x, y) = ye^x$

51-54 Use a computer to graph the function using various domains and viewpoints. Get a printout of one that, in your opinion, gives a good view. If your software also produces level curves, then plot some contour lines of the same function and compare with the graph. #s 51-2 NA

51. $f(x, y) = e^{-x^2} + e^{-2y^2}$ **52.** $f(x, y) = (1 - 3x^2 + y^2)e^{1-x^2-y^2}$

$$\frac{1}{e^{x^2}} + \frac{1}{e^{2y^2}}$$

