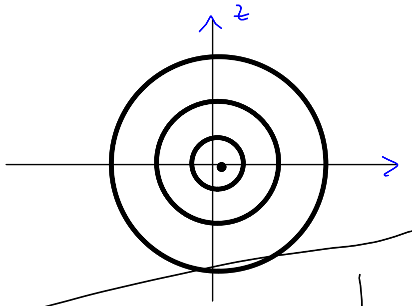
$x^2 - y^2 + z^2 = 1$

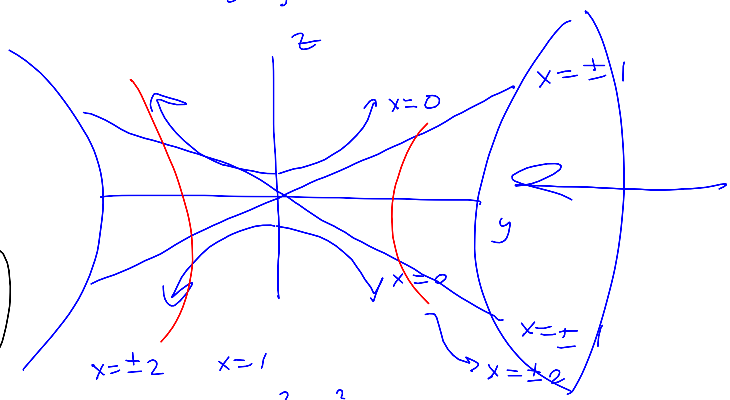
$z^2 - y^2 = 1$

$x^2 + z^2 = y^2 + 1$

$z^2 - y^2 = 1 - x^2$



Wolfram Cloud.



$z^2 - y^2 = 0$

$x = \pm 3$

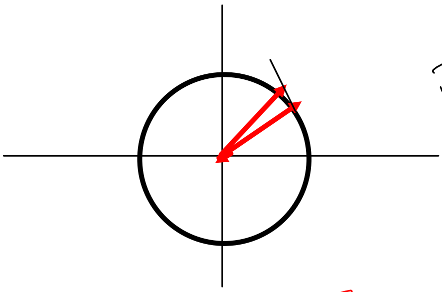
$y^2 = z^2$

$y^2 - z^2 = 8$

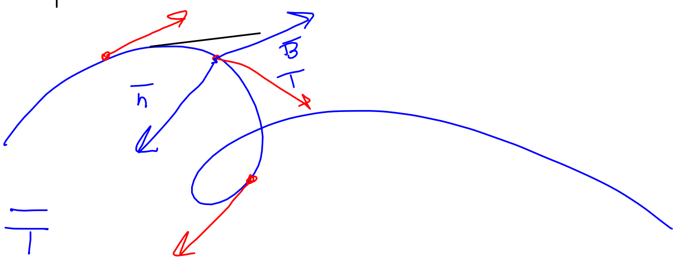
$x = \pm 2$

$y^2 - z^2 = 1$

unit Tangent $= \bar{T} = \dfrac{\bar{r}'}{\|\bar{r}'\|}$



These unit tangents have tangents that are $\perp$ to them



So

$\bar{T}' \perp \bar{T}$

Define $\bar{N} = \dfrac{\bar{T}'}{\|\bar{T}'\|}$ = unit Normal

Always points to the inside of the curve, because that's the direction $\bar{T}$ is changing towards.

$\bar{B}$ = Binormal $= \bar{T} \times \bar{N}$

Always "on top" of the direction of curvature

$\Rightarrow$ $\bar{N}$ is out the bottom

$\Rightarrow$ $\bar{B}$ is always to your left.

Recall $L = \int_a^b ds$

ds = increment of arc length.

Curve described by $\bar{r}(t)$.

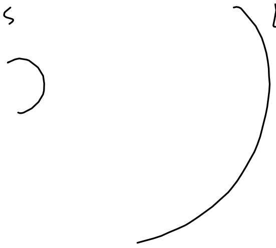
$= \sqrt{x'^2 + y'^2 + z'^2}\, dt$

$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$

$= \|\bar{r}'(t)\|\, dt$

Curvature: How much your direction changes with respect to a small change in arc length

Lots Less



$$\kappa = \left\| \frac{d\bar{T}}{ds} \right\|$$

$$= \left\| \frac{\frac{d\bar{T}}{dt}}{\frac{ds}{dt}} \right\| = \frac{\|\bar{T}'\|}{\|\bar{r}'\|} = \frac{\|\bar{r}'(t) \times \bar{r}''(t)\|}{\|\bar{r}'(t)\|^3}$$

There's a lengthy proof of this result

Theorem 10

$$x^2 - y^2 + z^2 - 2\cdot x + 2\cdot y + 4\cdot z + 2 = 0.$$

$$x^2 - 2x + 1^2 - y^2 + 2y + z^2 + 4z + 2^2 = -2 + 1 + 4$$

$$(x-1)^2 - (y^2 - 2y + 1^2) + (z+2)^2 = -2+1+4-1$$

$$(x-1)^2 - (y-1)^2 + (z+2)^2 = 2$$

$$(1, 1, -2)$$

65. Which of the following four planes are parallel? Are any of them identical?

Yes P_1: $4x - 2y + 6z = 3$

P_3: $-6x + 3y - 9z = 5$

Not ident.

P_2: $4x - 2y - 2z = 6$ NO

P_4: $z = 2x - y - 3$ NO

$-2x + y + z = -3$ SAME!

$-\frac{6}{4} = -\frac{3}{2}$ $\quad -\frac{3}{2}(-2) = 3 \quad -\frac{3}{2}(6) = 9$

$3(-\frac{3}{2}) = -\frac{9}{2} \neq 5$ Not equivalent.

P_1 $4\langle 1, -\frac{2}{4}, \frac{6}{4} \rangle$ Parallel

P_3 $-6\langle 1, -\frac{3}{6}, \frac{3}{2} \rangle$

Andrew's trick $\quad P_1, P_3 \parallel$

$P_2 \equiv P_4$