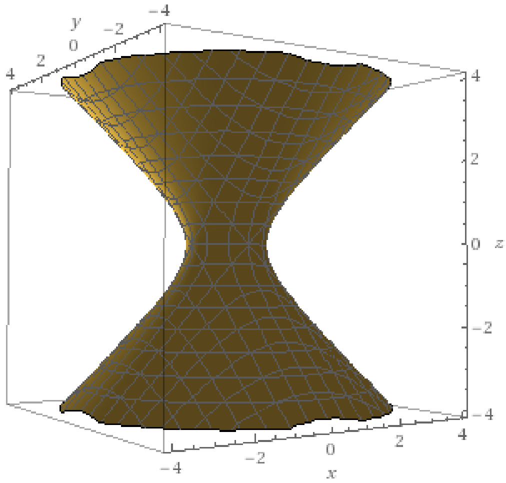
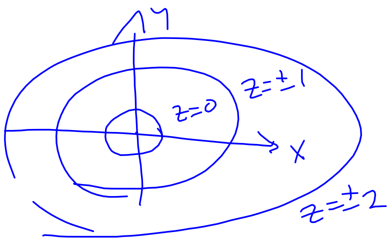

x
y
z
-4
-2
0
2
4


y
x
z=0
z=±1
z=±2

S12.3 # 14 Word problem.

$$\boxed{\begin{aligned} \text{Let } a &= \#\text{ of hamburgers sold} \\ b &= \text{'' '' hot dogs sold} \\ c &= \text{'' '' drinks sold} \end{aligned}}$$ Lexicon

$\frac{\text{cost}}{\text{burger}} = 2 \frac{\$}{\text{burger}}$, $\frac{\text{cost}}{\text{hotdog}} = 1.5 \frac{\$}{\text{hotdog}}$, $\frac{\text{cost}}{\text{drink}} = 1 \frac{\$}{\text{drink}}$.

If $\bar{A} = \langle a, b, c \rangle$ & $\bar{P} = \langle 2, 1.5, 1 \rangle$, then

$\bar{A} \cdot \bar{P} = 2a + 1.5b + 1c$ = cost of buying a meal.

$$\left(2 \frac{\$}{\text{hamburger}}\right)(a \text{ hamburgers}) = 2a \ \$$$

50. $2z = 4y - x, \quad 3x - 12y + 6z = 1$

$\Rightarrow \quad -x + 4y - 2z = 0$

$x - 4y + 2z = 0$

$\vec{n}_2 = \langle 3, -12, 6 \rangle = 3\langle 1, -4, 2 \rangle = \vec{n}_1$

The normals are multiples of one another $\Rightarrow$ parallel.

55–56 (a) Find parametric equations for the line of intersection of the planes and (b) find the angle between the planes.

55. $x + y + z = 1, \quad x + 2y + 2z = 1$

Finding intersection by elimination

$x + y + z = 1 \quad E1$

$x + 2y + 2z = 1 \quad E2$

$-E1 + E2$:

$-x - y - z = -1$

$x + 2y + 2z = 1$

$y + z = 0$

Goal: $\left[\begin{array}{ccc|c} 1 & a & b & c \\ 0 & 1 & d & e \\ 0 & 0 & 1 & f \end{array}\right]$

New System

$x + y + z = 1$

$y + z = 0$

Coefficient matrix:

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 1 \end{array}\right] \xrightarrow{-R1 + R2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right]$

Zeros under all the leading 1s, so we're done eliminating

Back-substitute:

This system is underdetermined (∞ # of sol'ns)

$x + y + z = 1$

$y + z = 0 \Rightarrow \boxed{y = -z}$

$x + (-z) + z = 1$

$\boxed{x = 1}$

$\Rightarrow$ Sol'n set is $\{(x, y, z) \mid x = 1, y = -z, z = \text{any real } \#\}$

$z \in \mathbb{R}$

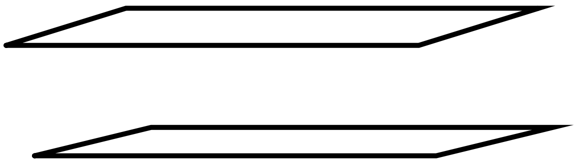
$y = -t, \ t \in \mathbb{R}$

We say "z is free"

Then let the parameter $t = z$

$x = 0, \ y = -t, \ z = t$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 3 & -1 & 2 \\ 3 & 6 & 9 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 3 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$



$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 3 & -1 & 2 \\ 1 & -1 & 3 & 1 \end{array}\right] \begin{array}{c} \sim \\ -2R1+R2 \\ -R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -1 & -7 & 0 \\ 0 & -3 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 21 & 0 \end{array}\right]$$

$x+2y+3z=1 \Rightarrow x=1$

$y+7z=0 \Rightarrow y=0$

$z=0$

$\{(1,0,0)\}$

36. $x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = 0$

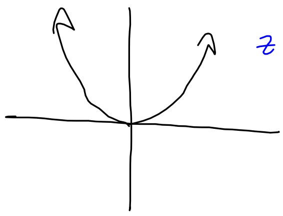
$x^2-2x \quad -y^2+2y \quad +z^2+4z \quad = -2$

$x^2-2x+1^2-(y^2-2y+1^2)+z^2+4z+2^2 = -2+1-1+4$

$(x-1)^2-(y-1)^2+(z+2)^2 = 2$

x

$y = x^2$ viewed as a surface in $\mathbb{R}^3$



$z = 0, \pm 1, \pm 2, \pm \ldots$