Today: Some linear algebra and some old-school analytic geometry.

16. (a) Find parametric equations for the line through (2, 4, 6) that is perpendicular to the plane $x - y + 3z = 7$.

(b) In what points does this line intersect the coordinate planes?

(a) $\bar{n} = \langle 1, -1, 3 \rangle \equiv \bar{v}$

Let $\bar{r}_0 = \langle 2, 4, 6 \rangle$

Then $\bar{r} = \bar{r}_0 + t\bar{v} \quad \forall t \in \mathbb{R}$

$\bar{r}(t) = \langle 2+t, 4-t, 6+3t \rangle$

Parametric Book Answer:

$$\boxed{x = t+2, \; y = -t+4, \; z = 3t+6}$$

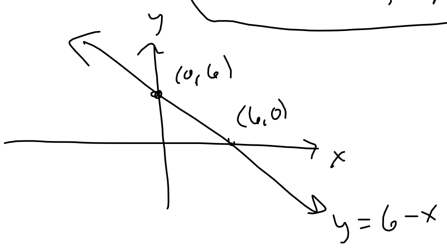
(b) xy-plane: $z = 0$

$\bar{r}(t) = \langle 2+t, 4-t, 0 \rangle$

$$\boxed{x = 2+t, \quad y = 4-t \quad t \in \mathbb{R}}$$

$x - 2 = t \Rightarrow y = 4 - (x-2) = 4 - x + 2 = 6 - x = y$

$$\boxed{\{ (x,y,0) \mid y = 6 - x, \; x \in \mathbb{R} \}}$$



But Stine: $z = 0$ means

$3t + 6 = 0 \Rightarrow$

$3t = -6$

$\Rightarrow t = -2$

$\Rightarrow$ Plug in $t = -2$ for x & y: Peanut Gallery Wins Again.

$x = 2 + (-2) = 0$

$y = 4 - (-2) = 6$

$(x, y, 0) = (0, 6, 0)$

yz-plane, xz-plane are similar.

17. Find a vector equation for the line segment from (2, −1, 4) to (4, 6, 1).

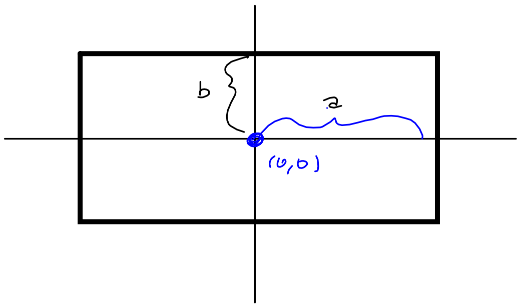
$\bar{r}(t) = (1-t)\langle 2,-1,4\rangle + t\langle 4,6,1\rangle \quad \forall\, t \in [0,1]$

Let $\bar{u} = \langle 2,-1,4\rangle$, $\bar{v} = \langle 4,6,1\rangle$. Then

$\bar{r}(t) = (1-t)\bar{u} + t\bar{v} \quad \forall\, t \in [0,1]$

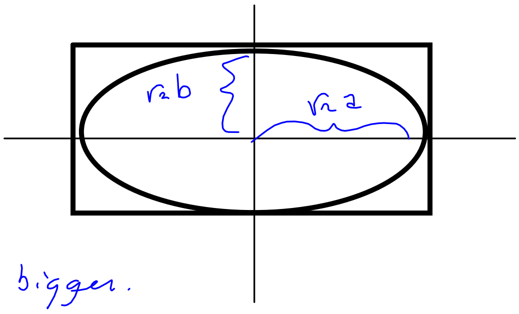
§12.6 Conic Sections Review

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$$

$$\frac{x^2}{2a^2} + \frac{y^2}{2b^2} = 1$$



Factor of $\sqrt{2}$ bigger.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 7$$

what about

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = -5$$

impossible!

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

To analyze these, we fix z (e.g.) & see what a cross-section of the surface.

See TABLE 1 §12.6

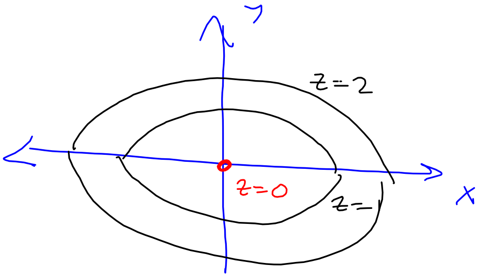
$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

0.

$z = 0$: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$

$\Longrightarrow x = y = 0$



$z = 1$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{c}$$

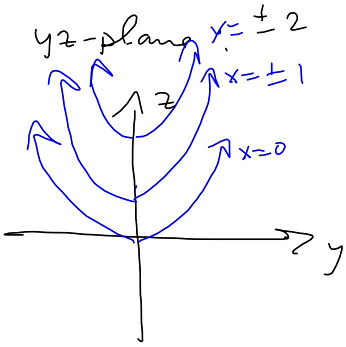
$$\frac{x^2}{\left(\frac{a^2}{c}\right)} + \frac{y^2}{\left(\frac{b^2}{c}\right)} = 1$$

$z = 2$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{2}{c}$$

$$\frac{x^2}{\left(\frac{2a^2}{c}\right)} + \frac{y^2}{\left(\frac{2b^2}{c}\right)} = 1$$

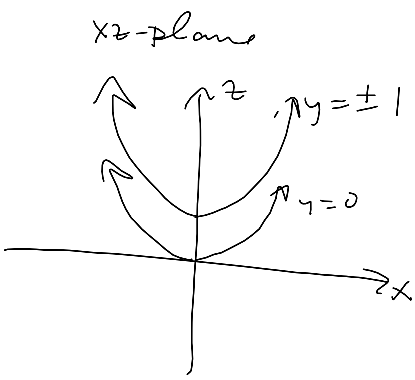
$z = -1$? Unpossible!



$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$z = \frac{y^2}{\frac{b^2}{c}} + \frac{x^2}{\frac{a^2}{c}}$$

$x = \pm 1$

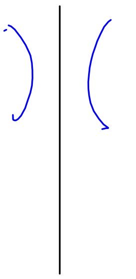


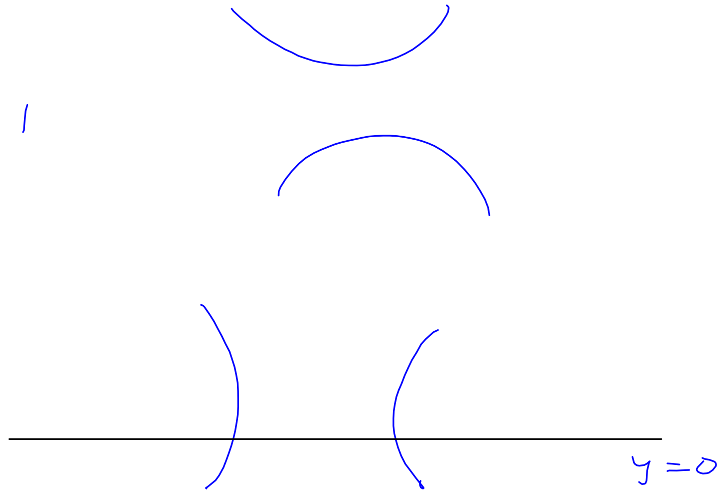
What's it look like in 3-D, based on these cross-sections, pieced together.

Hyperbola

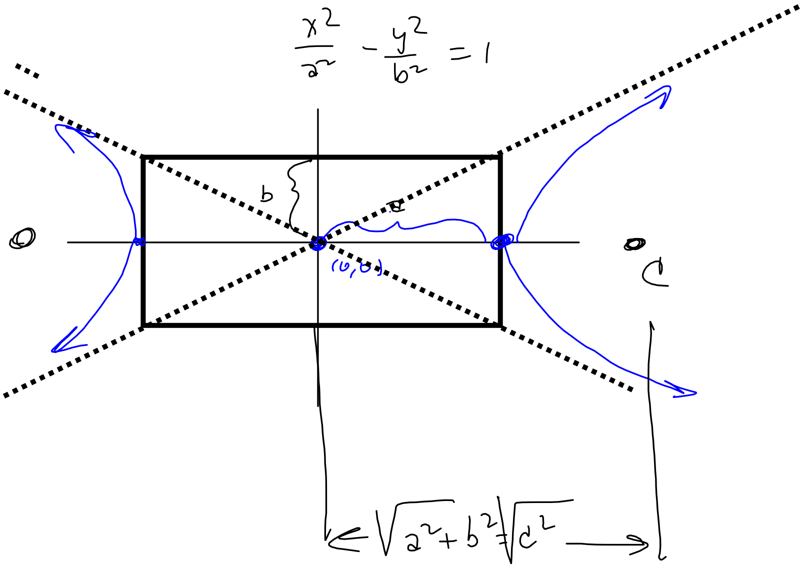
$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

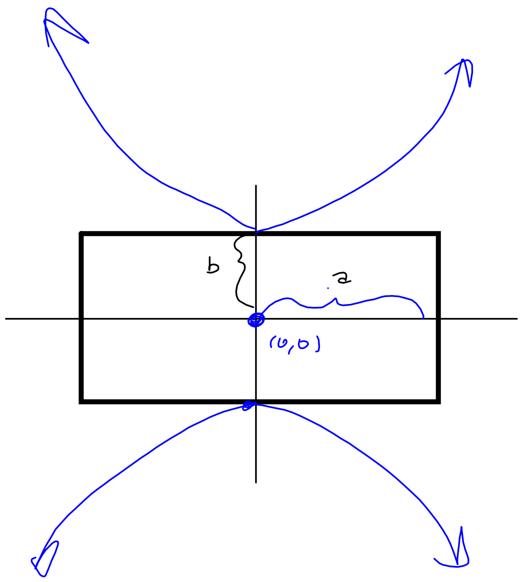
$x \neq 0$ even





$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$





$$\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$$