12.3 #41

12.4 #44

§12.5 will be in 2 parts

44. (a) Let P be a point not on the plane that passes through the points Q, R, and S. Show that the distance d from P to the plane is

$$d = \frac{|(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}|}{|\mathbf{a} \times \mathbf{b}|}$$

I'll do a video this afternoon/evening.

where $\mathbf{a} = \overrightarrow{QR}$, $\mathbf{b} = \overrightarrow{QS}$, and $\mathbf{c} = \overrightarrow{QP}$.

I'll make other videos for the theory that escaped us in §12.4.

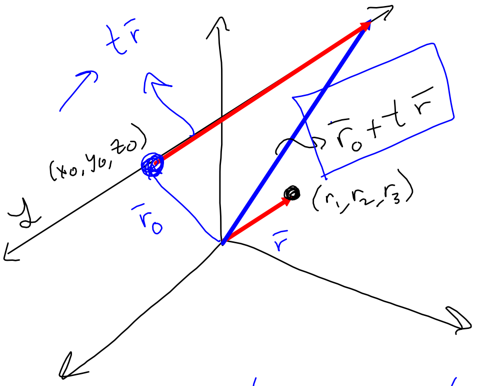
(b) Use the formula in part (a) to find the distance from the point $P(2, 1, 4)$ to the plane through the points $Q(1, 0, 0)$, $R(0, 2, 0)$, and $S(0, 0, 3)$.

Just plug into the dadgum formula

§13.5 Planes & Lines

A line is a point (given by a position vector $\bar{r}_0 = \langle x_0, y_0, z_0 \rangle$) plus all scalar multiples of a vector parallel to that line: $\bar{r}$.

$$\mathcal{L} : \{ \bar{v} \mid \bar{v} = \bar{r}_0 + t\bar{r},\ t \in \mathbb{R} \}$$



$\bar{r} = \langle r_1, r_2, r_3 \rangle$

So we get

$\bar{r}_0 + t\bar{r}$

$= \langle x_0, y_0, z_0 \rangle + t \langle r_1, r_2, r_3 \rangle$

$= \langle x_0 + tr_1, y_0 + tr_2, z_0 + tr_3 \rangle$

Parametric equations for $\mathcal{L}$

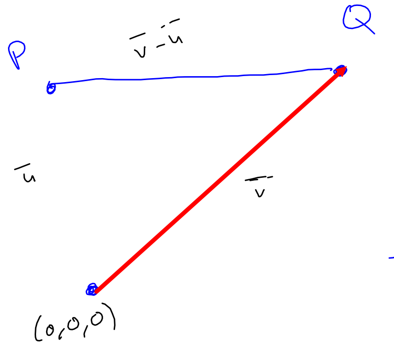
$$x = x_0 + tr_1, \quad y = y_0 + tr_2, \quad z = z_0 + tr_3$$

Symmetric Equations: (Solve parametric eq'ns for t)

$$t = \boxed{\frac{x - x_0}{r_1} = \frac{y - y_0}{r_2} = \frac{z - z_0}{r_3}}$$

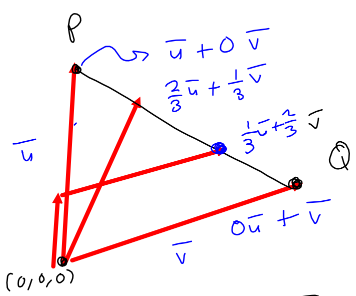
GOOD TO KNOW
NOT EVEN BAD IF YOU FORGET

Line segment between $P(x_0, y_0, z_0)$ & $Q(x_1, y_1, z_1)$



$\bar{u} = \langle x_0, y_0, z_0 \rangle$

$\bar{v} = \langle x_1, y_1, z_1$



$(1-t)\bar{u} + t\bar{v}$ for $t \in [0, 1]$

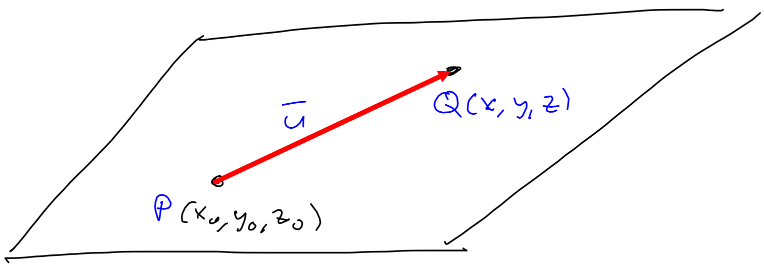
Line segment from $(2, 3, 1)$ to $(-1, 5, 7)$

$$\boxed{(1-t)\langle 2, 3, 1 \rangle + t\langle -1, 5, 7 \rangle}$$

POIFECK

A plane is defined by a point in the plane (x_0, y_0, z_0)
& a vector orthogonal to the plane. $\bar{n} = \langle a, b, c \rangle$.

Let (x, y, z) be on the plane.



$\bar{u}$ is "in" the plane

$\bar{u} = \langle x - x_0, y - y_0, z - z_0 \rangle$

Then $\bar{n} \cdot \bar{u} = \langle a, b, c \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle$

$= \boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0}$ → POIFECK

is the equation of the plane thru (x_0, y_0, z_0)
& orthogonal to $\bar{n}$.

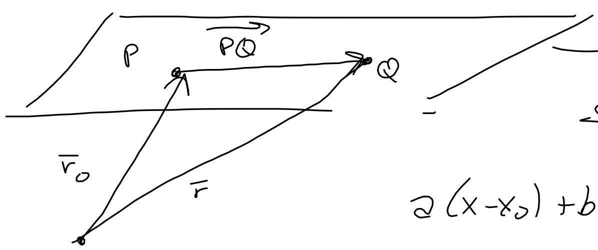
Book answers typically look like:

$ax + by + cz = d$, (where $d = ax_0 + by_0 + cz_0$

$\bar{n} = \langle a, b, c \rangle$ is NORMAL VECTOR

Plane containing $P(1, 3, 2)$ & $Q(3, -1, 6)$:

$\bar{r}_0 = \langle 1, 3, 2 \rangle$ $\quad \bar{r} = \langle 3, -1, 6 \rangle$



Continue Monday
~~Teacher's notes suck~~ we're outta time.

$a(x - x_0) + b(y - y_0) = 0$