§12.3 #42 video by day's

§12.2 #41

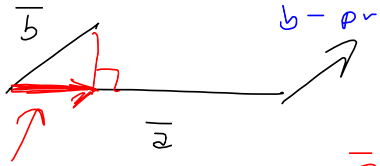
Show that $\text{orth}_{\bar{a}}\bar{b} = \bar{b} - \text{proj}_{\bar{a}}\bar{b}$ is orthogonal to $\bar{a}$

copy prob. correctly!

Recall $\cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{\|\bar{a}\|\|\bar{b}\|} = 0$, iff $\bar{a}\cdot\bar{b} = 0$

$\text{proj}_{\bar{a}}\bar{b} = \dfrac{|\bar{a}\cdot\bar{b}|}{\|\bar{a}\|^2}\bar{a} \longrightarrow$

$\bar{b}$



$\bar{a}$

$\text{proj}_{\bar{a}}\bar{b}$

$\bar{b} - \text{proj}_{\bar{a}}\bar{b} = \bar{b} - \dfrac{|\bar{a}\cdot\bar{b}|}{\|\bar{a}\|^2}\bar{a}$

Want this to be $\perp$ to $\bar{b}$:

$\bar{a}\cdot(\bar{b} - \text{proj}_{\bar{a}}\bar{b}) = 0$

$= \bar{a}\cdot\bar{b} - \bar{a}\cdot\text{proj}_{\bar{a}}\bar{b}$

$= \bar{a}\cdot\bar{b} - \bar{a}\cdot\dfrac{|\bar{a}\cdot\bar{b}|}{\|\bar{a}\|^2}\bar{a} = \bar{a}\cdot\bar{b} - |\bar{a}\cdot\bar{b}|$

$= 0?$

$\bar{a} \cdot \bar{b} = \langle a_1, a_2, a_3 \rangle \cdot \langle b_1, b_2, b_3 \rangle = a_1 b_1 + a_2 b_2 + a_3 b_3$

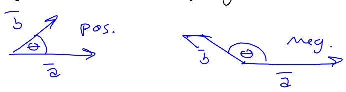
$= \sum_{k=1}^{3} a_k b_k$

Σ is nice, because here's the statement for

$\bar{a}, \bar{b} \in \mathbb{R}^{10} \Rightarrow \bar{a} \cdot \bar{b} = \sum_{k=1}^{10} a_k b_k$

So we can extend these ideas to higher dimensions in very elegant fashion.

$\text{comp}_{\bar{a}} \bar{b}$ = scalar projection = $\dfrac{\bar{a} \cdot \bar{b}}{\|\bar{a}\|}$



$\bar{a} \perp \bar{b}$ iff $\bar{a} \cdot \bar{b} = 0$

$f(x) = x - 1$ $\qquad f : \mathbb{R} \longrightarrow \mathbb{R}$

$\cdot : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$

$\cdot(\bar{a}, \bar{b}) = \cdot\Big(\underbrace{(a_1, a_2, a_3), (b_1, b_2, b_3)}_{\text{is in } \mathbb{R}^3 \times \mathbb{R}^3}\Big) = \sum_{k=1}^{3} a_k b_k \in \mathbb{R}$

Cartesian product of $\mathbb{R}^3$ with itself.

Distance : $\mathbb{R}^2 \times \mathbb{R}^2 \longrightarrow \mathbb{R}$

$A(x_1, y_1)$, $B(x_2, y_2)$

$D(A, B) = D\big((x_1, y_1), (x_2, y_2)\big) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \in \mathbb{R}$

New! $\quad$ Cross Product of 2 vectors

$\times : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

Recall Determinants:

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (1)(4) - (3)(2) = -2$

$B = \begin{bmatrix} -1 & 2 \\ 5 & -7 \end{bmatrix} \Rightarrow |B| = 7 - 10 = -3$

Expansion by minors.

$A = \begin{bmatrix} 1^{+} & 2^{-} & 3^{+} \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \Rightarrow |A| = +1(5)(9) - (8)(6) - \Big(2\big((4)(9) - (7)(6)\big)\Big)$

$+ 3\big((4)(8) - (7)(5)\big) = 45 - 48 - 2(36 - 42)$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $\quad -2\big((4)(9) - (7)(6)\big)$

$+ 3(56 - 35)$

$= -3 - 2(-6) + 3(21)$

$= -3 + 12 + 63$

$= 75 - 3 = \boxed{72 = |A|}$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $\quad 3\big((4)(8) - (7)(5)\big)$

Let $\bar a = \langle a_1, a_2, a_3 \rangle$, $\bar b = \langle b_1, b_2, b_3 \rangle$

Then $\bar a \times \bar b$ is the determinant of this matrix:

$$\begin{bmatrix} \bar i\,+ & \bar j\,- & \bar k\,+ \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$

, where the $\bar i, \bar j, \bar k$ are symbolic

$$= \bar i\,(a_2 b_3 - a_3 b_2) - \bar j\,(a_1 b_3 - a_3 b_1) + \bar k\,(a_1 b_2 - a_2 b_1)$$

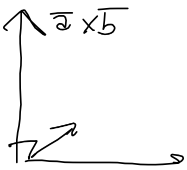
$$= \langle a_2 b_3 - a_3 b_2, -a_1 b_3 + a_3 b_1, a_1 b_2 - a_2 b_1 \rangle$$

$\bar a \times \bar b$ in practice:

$$\begin{array}{l} \langle a_1, a_2, a_3 \rangle\ a_1, a_2 \\ \langle b_1, b_2, b_3 \rangle\ b_1, b_2 \\ \hline \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle \end{array}$$

$\bar a = \langle 1, 2, 3 \rangle$

$\bar b = \langle 4, 5, 3 \rangle$



$$\begin{array}{lll} & \bar a & \langle 1, 2, 3 \rangle\ 1, 2 \\ \times & \bar b & \langle 4, 5, 3 \rangle\ 4, 5 \\ \hline \end{array}$$

$\bar w = \langle 6-15, 12-3, 5-8 \rangle = \langle -9, 9, -3 \rangle = \bar a \times \bar b \in \mathbb{R}^3$

$\bar a \cdot \bar w = \langle 1, 2, 3 \rangle \cdot \langle -9, 9, 3 \rangle = -9 + 18 + 9 = 0$

$\bar b \cdot \bar w = \langle 4, 5, 3 \rangle \cdot \langle -9, 9, 3 \rangle = -36 + 45 + 9 = 0$

$\boxed{\text{TORQUE}}$