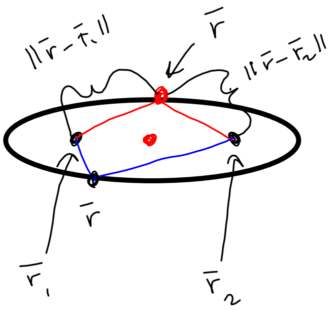
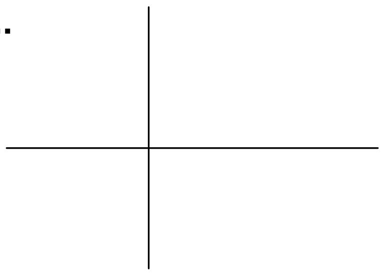
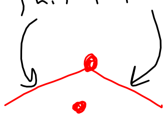
§12.2 #42

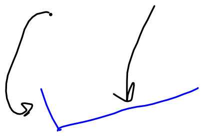
$$\|\bar{r}-\bar{r}_1\| + \|\bar{r}-\bar{r}_2\| = K$$





This + That = This + That $= K > \|\bar{r}_1-\bar{r}_2\|$





If $K = \|\bar{r}_1-\bar{r}_2\|$, then the ellipse is just a line segment.

If $K < \|\bar{r}_1-\bar{r}_2\|$, we endup with a false statement.

because then $\|\bar{r}-\bar{r}_1\| + \|\bar{r}-\bar{r}_2\| > K$!

Proof by triangle inequality.

$$|a|+|b| \geq |a+b|$$

$$\|\bar{r}-\bar{r}_1\| + \|\bar{r}-\bar{r}_2\|$$

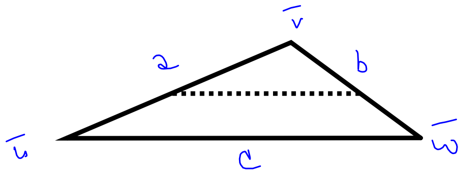
$$\geq \|\bar{r}-\bar{r}_1+\bar{r}-\bar{r}_2\| = \|2\bar{r}-\bar{r}_1-\bar{r}_2\|$$

$$\ldots > K\,?!$$

but teacher forgot the proof.

§12.2 #45

(45) We use vectors to prove that the line joining the midpoints of 2 sides of a triangle is parallel to the 3^{rd} side & half its length.



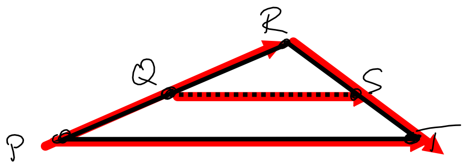
lengths a, b, c

vertices $\bar{u}, \bar{v}, \bar{w}$

$a = \|\bar{v} - \bar{u}\|$

$b = \|\bar{v} - \bar{w}\|$

$c = \|\bar{w} - \bar{v}\|$



Want to show

$\|\overrightarrow{QS}\| = \frac{1}{2}\|\overrightarrow{PT}\|$

$\|PQ\| = \frac{1}{2}\|\overrightarrow{PR}\| = \|\overrightarrow{QR}\|$

$\|\overrightarrow{RS}\| = \frac{1}{2}\|\overrightarrow{RT}\| = \|\overrightarrow{ST}\|$

Want to show $\overrightarrow{QS} \parallel \overrightarrow{PT}$

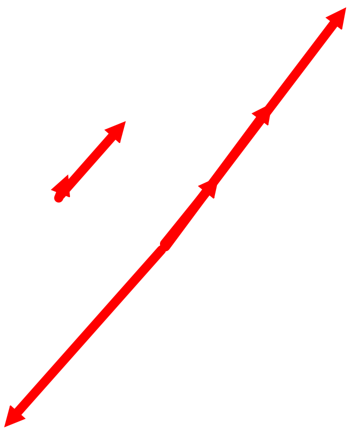
$\overrightarrow{QS} = \bar{u}$

$\overrightarrow{PT} = \bar{v}$

Parallel means:

$\bar{u} = k\bar{v}$ for some $k \in \mathbb{R}$

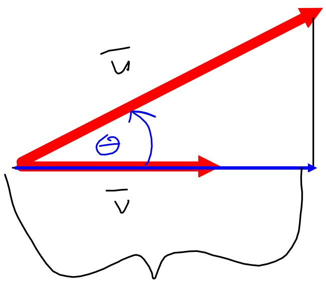
This is the kind of thing you do when parsing these kinds of questions.



This is pretty abstract. Don't waste TOO much time, when there's somebody to ask.

Recall $\cos\theta = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\,\|\bar{v}\|}$

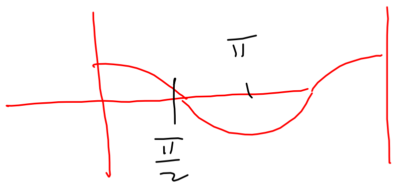
Want the length of the projection of $\bar{u}$ onto $\bar{v}$.

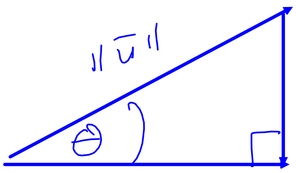


$a = \text{comp}_{\bar{v}}\,\bar{u}$

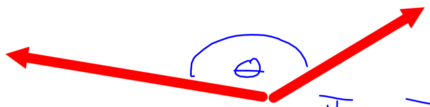
= component of $\bar{u}$ in the direction of $\bar{v}$.

= signed length.





$\text{comp}_{\bar{v}}\,\bar{u}$



$\theta > \frac{\pi}{2} \Rightarrow \text{comp}_{\bar{v}}\,\bar{u} < 0$

$$\frac{\text{comp}_{\bar{v}}\,\bar{u}}{\|\bar{u}\|} = \cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\,\|\bar{v}\|}$$

$$\Rightarrow \text{comp}_{\bar{v}}\,\bar{u} = \|\bar{u}\|\cos\theta = \frac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|} = \text{comp}_{\bar{v}}\,\bar{u}$$

Now, let's talk about the PROJECTION of $\bar{u}$ onto $\bar{v}$

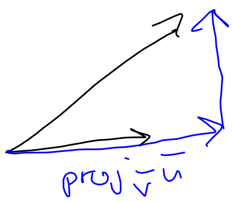
$\text{proj}_{\bar{v}}\bar{u}$ is the vector in the direction of $\bar{v}$ that has length equal to $|\text{comp}_{\bar{v}}\bar{u}|$, i.e.,

$$\text{proj}_{\bar{v}}\bar{u} = \underbrace{|\text{comp}_{\bar{v}}\bar{u}|}_{\text{size}}\,\underbrace{\frac{\bar{v}}{\|\bar{v}\|}}_{\text{Direction}} = \left|\frac{\bar{u}\cdot\bar{v}}{\|\bar{v}\|}\right|\frac{\bar{v}}{\|\bar{v}\|}$$

$$= \frac{|\bar{u}\cdot\bar{v}|}{\|\bar{v}\|^2}\,\bar{v}$$

$$= \text{proj}_{\bar{v}}\bar{u}$$

$\text{orth}_{\bar{v}}\bar{u}$ = orthogonal component



$\text{orth}_{\bar{v}}\bar{u} = \bar{u} - \text{proj}_{\bar{v}}\bar{u}$

This is THE THEORY behind Gram-Schmidt orthogonalization process,

that you will see in Linear Algebra.

Dot product is not "times" because it maps from R3XR3 into R.

"Times" would map straight into R3.

$+ : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$

$3+7 = 10 \in \mathbb{R}$

But

$\cdot : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$

$\overline{u}\,\overline{v} = \overline{w} \in \mathbb{R}^3$