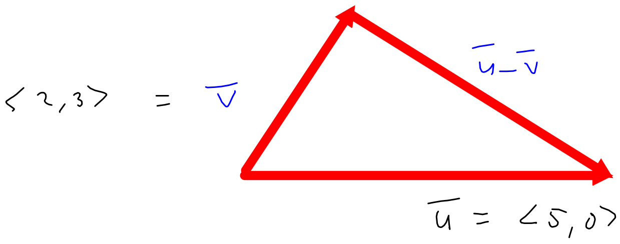
Some 12.2 Stuff

$\bar{u} - \bar{v}$




$\bar{u} - \bar{v} = \langle 3,-3 \rangle = \bar{w}$

Magnitude of $\bar{u} = \sqrt{u_1^2 + u_2^2 + u_3^2}$ in 3-D

Unit vector in the direction of $\bar{u}$:

$$\frac{1}{\|\bar{u}\|}\bar{u} = \frac{\bar{u}}{\|\bar{u}\|}$$

$$\bar{u} = \langle 1,2,3 \rangle \Rightarrow \|\bar{u}\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$$

$$\Rightarrow \frac{\bar{u}}{\|\bar{u}\|} = \frac{\langle 1,2,3 \rangle}{\|\bar{u}\|} = \frac{1}{\|\bar{u}\|}\bar{u} = \frac{1}{\sqrt{14}}\langle 1,2,3 \rangle$$

$$\& \left\| \frac{\bar{u}}{\|\bar{u}\|} \right\| = \left\| \left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle \right\|$$

$$= \sqrt{\left(\frac{1}{\sqrt{14}}\right)^2 + \left(\frac{2}{\sqrt{14}}\right)^2 + \left(\frac{3}{\sqrt{14}}\right)^2} = \sqrt{\frac{1+4+9}{14}} = \sqrt{\frac{14}{14}} = 1$$

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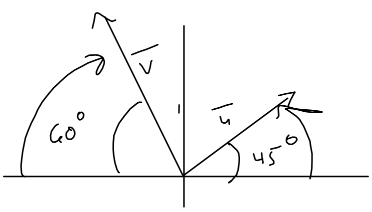
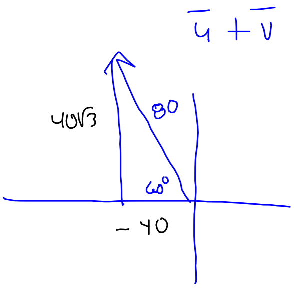
Application of $\bar{u} + \bar{v}$:

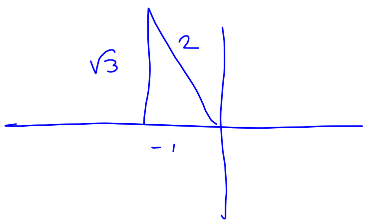



$\|\bar{v}\| = 80$ lbs

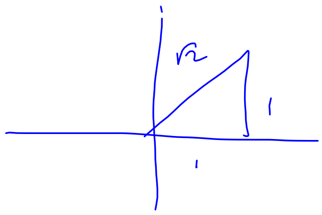
$\|\bar{u}\| = 50$ lbs

what's the net force & direction of the force on dear old Dad?



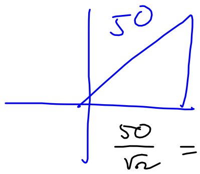



$\bar{v} = \langle -40, 40\sqrt{3} \rangle$




$\sqrt{2}\, x = 50$

$x = \frac{50}{\sqrt{2}}$




$\frac{50}{\sqrt{2}} = 25\sqrt{2}$

$\frac{50}{\sqrt{2}} = 25\sqrt{2}$

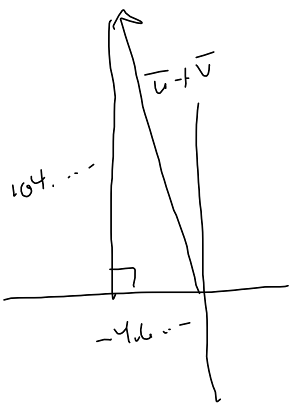
$\bar{u} = \langle 25\sqrt{2}, 25\sqrt{2} \rangle$

$\Rightarrow \quad \bar{u} + \bar{v} = \langle 25\sqrt{2} - 40, 25\sqrt{2} + 40\sqrt{3} \rangle$

$\approx \langle -4.64466094, 104.6373714 \rangle$

Horizontal component of the force.

Vertical component of the force



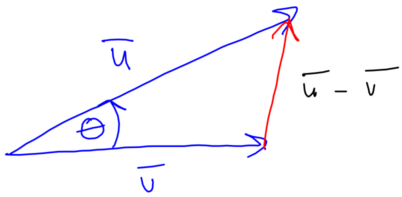
13.3 Dot Product

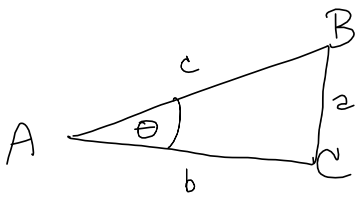
$\bar{u} = \langle u_1, u_2, u_3 \rangle$ , $\bar{v} = \langle v_1, v_2, v_3 \rangle \Rightarrow$

$\bar{u} \circ \bar{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

$\bar{u} \circ \bar{u} = u_1 u_1 + u_2 u_2 + u_3 u_3 = u_1^2 + u_2^2 + u_3^2 = \|\bar{u}\|^2$

Recall $\|\bar{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$




$(a-b)^2 = a^2 - 2ab + b^2$

Law of cosines

$a^2 = b^2 + c^2 - 2bc \cos A$

$\|\bar{u} - \bar{v}\|^2 = \|\bar{u}\|^2 + \|\bar{v}\|^2 - 2\|\bar{u}\|\|\bar{v}\| \cos\theta$

$\|\bar{u}\|^2 = \bar{u} \circ \bar{u} = u_1^2 + u_2^2 + u_3^2$

$\|\bar{v}\|^2 = \bar{v} \cdot \bar{v} = v_1^2 + v_2^2 + v_3^2$

$\|\bar{u} - \bar{v}\|^2 = (\bar{u} - \bar{v}) \cdot (\bar{u} - \bar{v}) = \langle u_1 - v_1, u_2 - v_2, u_3 - v_3 \rangle \circ \langle u_1 - v_1, u_2 - v_2, u_3 - v_3 \rangle$

$= (u_1 - v_1)^2 + (u_2 - v_2)^2 + (u_3 - v_3)^2$

$= u_1^2 - 2u_1 v_1 + v_1^2 + u_2^2 - 2u_2 v_2 + v_2^2 + u_3^2 - 2u_3 v_3 + v_3^2$

$$u_1^2 - 2u_1v_1 + v_1^2 + u_2^2 - 2u_2v_2 + v_2^2 + u_3^2 - 2u_3v_3 + v_3^2$$

$$= u_1^2 + u_2^2 + u_3^2 + v_1^2 + v_2^2 + v_3^2 - 2\|\bar{u}\|\|\bar{v}\|\cos A$$

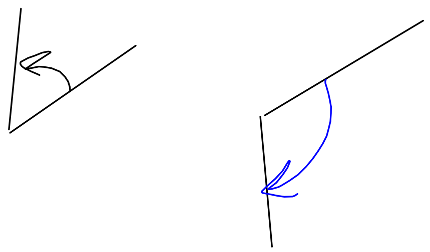
$$-2u_1v_1 - 2u_2v_2 - 2u_3v_3 = -2\|\bar{u}\|\|\bar{v}\|\cos A$$

$$-2\underbrace{(u_1v_1 + u_2v_2 + u_3v_3)} = -2\|\bar{u}\|\|\bar{v}\|\cos A$$

$$\bar{u} \cdot \bar{v} = \|\bar{u}\|\|\bar{v}\|\cos A$$

$$\boxed{\cos A = \frac{\bar{u} \cdot \bar{v}}{\|\bar{u}\|\|\bar{v}\|}}$$ Never forget!

A will always be between $0^\circ$ & $180^\circ$



Direction Angles

2 problems.