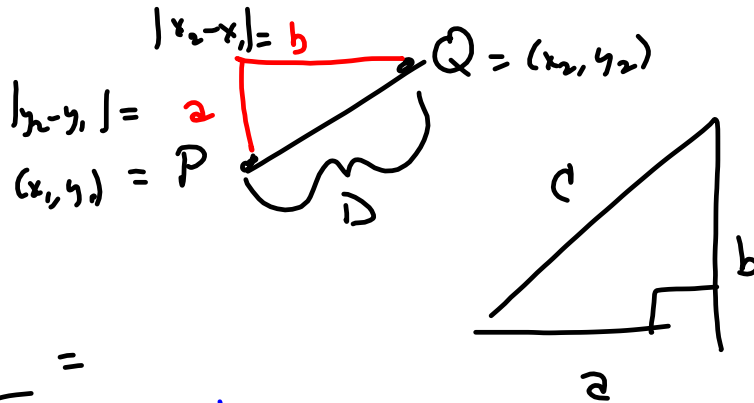


Distance :  $P(x_1, y_1)$   $Q(x_2, y_2)$



$$\sqrt{c^2} = |c|$$

$$\sqrt{a^2 + b^2} = \sqrt{c^2} = |c|$$

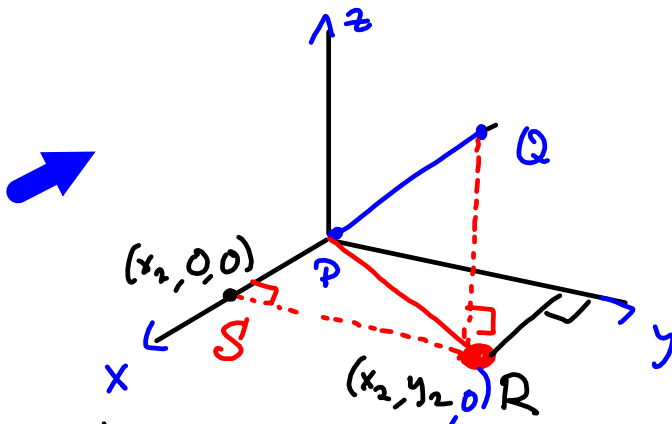
$$D = \sqrt{a^2 + b^2}$$

$$= \sqrt{|y_2 - y_1|^2 + |x_2 - x_1|^2}$$

$$= \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$c = \pm \sqrt{a^2 + b^2}$$

$\Rightarrow c = \sqrt{a^2 + b^2}$  if we assume the length is positive.



Right-hand rule.

$$P^*(x_1^*, y_1^*, z_1^*)$$

$$Q^*(x_2^*, y_2^*, z_2^*)$$

Distance in 3-D :

$$|\vec{PR}| = \sqrt{|\vec{PS}|^2 + |\vec{SR}|^2}$$

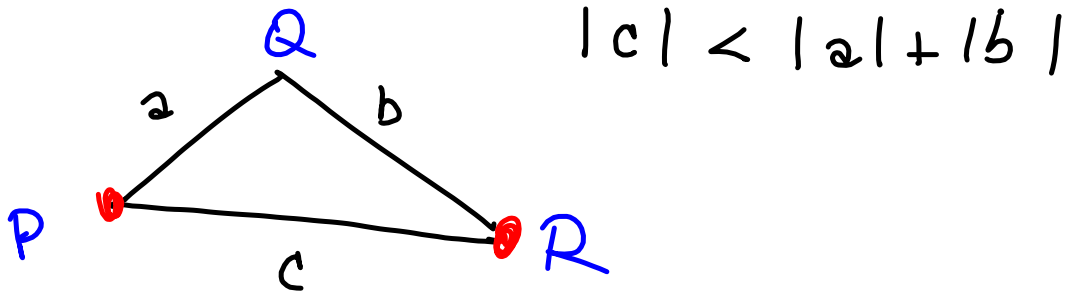
$$P(0, 0, 0)$$

$$Q(x_2, y_2, z_2)$$

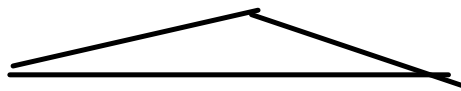
$$|PQ| = \sqrt{|PR|^2 + |RQ|^2}$$

$$= \sqrt{|\vec{PS}|^2 + |\vec{SR}|^2 + |\vec{RQ}|^2}$$

$$= \sqrt{(x_2 - 0)^2 + (y_2 - 0)^2 + (z_2 - 0)^2}$$



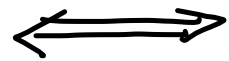
In general,  $|c| \leq |a| + |b|$   
 Only "=" when Q & R  
 lie on the same line.



This is the  
triangle inequality.



if and only if



Q & R lie on the SAME line iff

$$|\vec{PQ}| + |\vec{QR}| = |\vec{PR}|$$

(otherwise

See S 12.1 #9

$$|a| + |b| > |c|)$$

A circle is the set of all points equidistant from a fixed point (called the "center") and the distance in question is called the "radius."

Let  $(x_1, y_1, z_1)$  be the center  
and let  $(x, y, z)$  be any point  
and  $r =$  the radius.

Then  $r = \sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2}$

i.e.,  $(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2 = r^2$   
is the equation of a SPHERE.

Completing the Square Skill:

$$x^2 + y^2 + z^2 - 6x + 4y - 2z = 11$$

Show that this is a sphere.  
State its center & radius.

$$\begin{aligned} x^2 - 6x + 3^2 + y^2 + 4y + 2^2 + z^2 - 2z + 1^2 &= 11 + 9 + 4 + 1 \\ (x-3)^2 + (y+2)^2 + (z-1)^2 &= 25 = 5^2 \\ (h, k, w) = \text{center} &= (3, -2, 1) \\ \& r = 5 \end{aligned}$$

$P(2, 3, 4)$  has position vector

$$\vec{v} = \langle 2, 3, 4 \rangle$$

has size  
& direction

