MAT 203-G11 | Test 2 – Spring, 2016 | Name KGY

Covers Chapter 14 | 75 Points and 75% of Test 2 Grade

Please, everyone follow the homework drill, except for folding. Plus EXTRA neatness and EXTRA space for max points.

1. (5 pts) Find and graph the domain of the function $f(x,y)=\dfrac{\sqrt{x^2+y^2-25}}{\ln(x-y)}$.

2. **Bonus** (5 pts) For the person who studied the old test's #2, anyway…

   Use the fact that $\sqrt{x^2+y^2} \ge \sqrt{x^2}$ to show that $\lim\limits_{(x,y)\to(0,0)}\left(\dfrac{xy}{\sqrt{x^2+y^2}}\right)=0$.

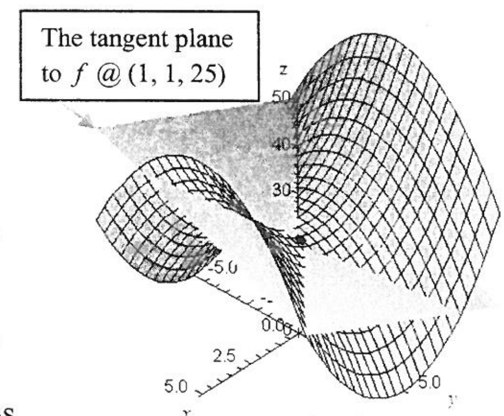
3. Find the 1st partials, f_x and f_y, for …

   a. (5 pts) … $f(x,y)=\cos(x^2y^2)$

   b. (5 pts) … $f(x,y)=\int_y^x \xi^3 \arctan(\xi^4)\,d\xi$. Keep your eye where it matters! FTC I, baby!

4. (5 pts) Use implicit differentiation to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ for the equation: $yz + y\ln(x) = z^2$ ..

5. Let $f(x,y)=x^2-y^2+25$.



$f(x,y)=x^2-y^2+25$

   a. (5 pts) Find an equation of the tangent plane to $f(x,y)$ at the point $(1, 1, 25)$. Get z all by itself in that equation, and you'll have the linearization, $z=L(x,y)$, that will give us the tangent plane approximations we ask for, next! :o)

   b. (5 pts) Use your previous answer to approximate $f(1.2, 1)$. If you drew a blank on part 'a', make one up, as long as it contains $(1,1,25)$ and it's not horizontal or vertical.

   c. (5 pts) Find the *actual* value of $f(1.2, 1)$.

   d. (5 pts) Find Δz for the change in z, from $f(1,1)=25$ to $f(1.2, 1)$.

   e. (5 pts) Find the differential estimate, dz, to the actual change, Δz, that you found in the previous problem. You can do this by forming the differential dz, or you can just do some subtracting, using earlier work.

6. Suppose the function $f(x,y)=x^2+4y^2-50$ describes an elliptical sinkhole in the middle of downtown Greeley (Pick your least favorite business.).

   a. (5 pts) What is the gradient at the point $(2,3,-10)$?

   b. (5 pts) What is the directional derivative $D_{\overline{u}}$ in the direction of $\overline{u}=\langle 2,1\rangle$?

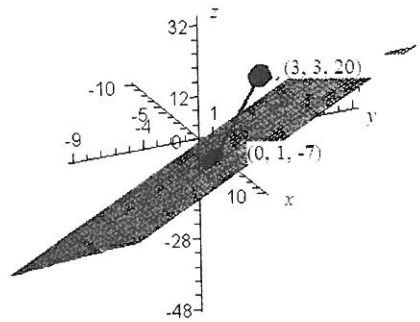
7. Answer one of the following. You probably already know the answer to both. What I want to see is your use of calculus to find the point in question.

   a. (5 pts) Find critical values and use the 2nd derivative test to find the saddle point on the surface $f(x,y)=x^2-y^2+25$ from #5. It's almost trivial, but go through all the motions, K?

   b. (5 pts) Find critical values and use the 2nd derivative test to find the minimum point on the sinkhole from #6. I know! I know! But go through all the motions, OK?

8. (5 pts) Find the distance from the point (3, 3, 20) to the plane $2x+2y-z=9$.

9. (5 pts) Find the point on the plane in #8 that is closest to (3, 3, 20).



Work any two of the following:

Bonus (5 pts) Find f_τ and f_ω if $f(\tau,\omega)=\int_{\sin(\tau)}^{\cos(\omega)}\pi\left(x^2e^{\pi x}\right)dx$. FTC I with Chain Rule!

Bonus (5 pts) Find the 1st partials, f_x and f_y, for $f(x,y)=y\int_0^x\xi^3\arctan\left(\xi^4\right)d\xi$. FTC I with Product Rule!

Bonus (5 pts) Just in case you wanted to show me your triangle play, or you happen to have the derivative of $\arccos(x)$ handy, or can remember it:

Find the 1st partials, f_x and f_y, for $f(x,y)=\arccos\left(x^2y^2\right)$

203 E2

(1) $x^2+y^2-25 \geq 0$

$x^2+y^2 \geq 25$

Exterior of disk, radius $r=5$ center $=(0,0)$

$x-y>0$

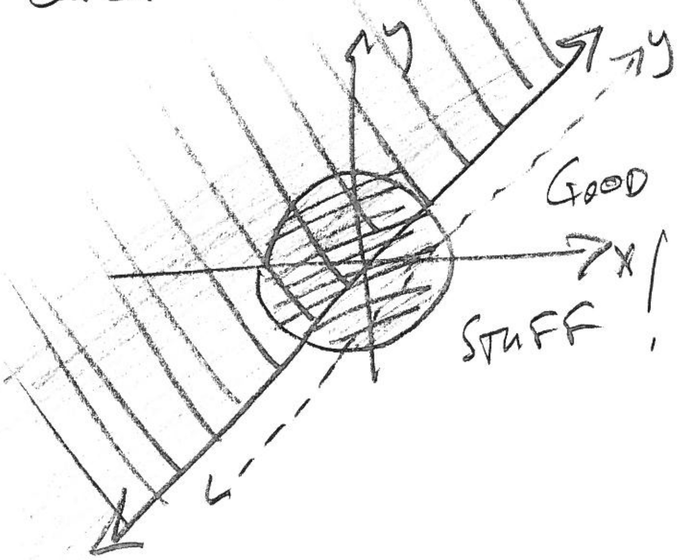
$x>y$

$y<x$

under the line

5pts

5pts



$\ln(x-y) \neq 0$

So $x-y \neq 1$

so $y \neq x-1$

(2B) $\left| \frac{xy}{\sqrt{x^2+y^2}} - 0 \right| = \left| \frac{xy}{\sqrt{x^2+y^2}} \right| \leq \left| \frac{xy}{\sqrt{x^2}} \right|$ 5pts

$= \left| \frac{xy}{|x|} \right| = \left| \frac{xy}{x} \right| = |y| \xrightarrow{(x,y)\to(0,0)} 0$ ☑

(3)(a) $f(x,y) = \cos(x^2y^2) \implies \boxed{\begin{aligned} f_x &= -2xy^2 \sin(x^2y^2) \\ f_y &= -2x^2y \sin(x^2y^2) \end{aligned}}$ 5pts

(3b) $f(x,y) = \int_y^x \xi^3 \arctan(\xi^4)\, d\xi = -\int_x^y \xi^3 \arctan(\xi^4)\, d\xi$ 5pts

$\implies \boxed{f_x = x^3 \arctan(x^4)}$, $\boxed{f_y = -y^3 \arctan(y^4)}$

203 E2

(4) $yz + y\ln(x) = z^2$

$\frac{d}{dx}\left[\qquad \right]$

5pb

$\Rightarrow yz' + \frac{y}{x} = 2zz'$, where $z' \equiv \frac{dz}{dx}$

$\Rightarrow yz' - 2zz' = -\frac{y}{x}$

$\Rightarrow z'(y-2z) = -\frac{y}{x}$

$\Rightarrow z' = \boxed{-\frac{y}{x(y-2z)} = \frac{dz}{dx}} = \frac{y}{x(2z-y)}$

Define: $z' \equiv \frac{dz}{dy}$

$\frac{d}{dy}\left[\text{original} \right]$

$z + yz' + \ln(x) = 2zz'$

$yz' - 2zz' = -\ln x - z$

$z'(y-2z) = -\ln x - z$

$z' = \boxed{\frac{-\ln x - z}{y-2z} = \frac{dz}{dy}} = \frac{\ln x + z}{2z-y}$

203 E2

$f_x = 2x, \quad f_y = -2y$

(5) $f(x,y) = x^2 - y^2 + 25$

(a) Tan. Plane @ $(1,1,25)$:

$f_x(1,1)(x-1) + f_y(1,1)(y-1) + 25 = L(x,y)$

5 pts

$= 2(1)(x-1) - 2(1)(y-1) + 25$

$= \boxed{2(x-1) - 2(y-1) + 25 = L(x,y)}$

5 pts

(b) $f(1.2,1) \approx L(1.2,1) = 2(.2) + 25 = \boxed{25.4 = L(1.2,1)}$

(c) $f(1.2,1) = (1.2)^2 - 1^2 + 25$

$= 1.44 - 1 + 25 = \boxed{25.44 = f(1.2,1)}$

5 pts

(d) $\Delta z = f(x_1,y_1) - f(x_0,y_0) = f(1.2,1) - f(1,1)$

$= 25.44 - 25 = \boxed{.44 = \Delta z}$

5 pts

(e) $dz = L(1.2,1) - f(1,1) = 25.4 - 25 = \boxed{.4 = dz}$

5 pts

203 E2

(6) $f(x,y) = x^2 + 4y^2 = 50.$

(a) $\nabla f(2,3)$:

$\nabla f = \langle 2x, 8y \rangle$

$\nabla f(2,3) = \langle 2(2), 8(3) \rangle = \boxed{\langle 4, 24 \rangle}$

5pts

(b) $\bar{u} = \langle 2, 1 \rangle \Rightarrow \frac{\bar{u}}{|\bar{u}|} = \frac{1}{\sqrt{5}} \langle 2, 1 \rangle$

5pts

$\Rightarrow D_{\bar{u}} = \frac{1}{\sqrt{5}} \langle 2, 1 \rangle \cdot \langle 4, 24 \rangle$

$= \frac{1}{\sqrt{5}} (8 + 24) = \boxed{\frac{32}{\sqrt{5}} = \frac{32\sqrt{5}}{5}}$

STUDENTS DO ONE :

(7) (a) $f(x,y) = x^2 - y^2 + 25 \Rightarrow$

$f_x = 2x \overset{\text{SET}}{=} 0 \Rightarrow x = 0$

$f_y = -2y \overset{\text{SET}}{=} 0 \Rightarrow y = 0$

$\Big\}$ $(0,0)$

$f_{xx} = 2$

$f_{yy} = -2$

$f_{xy} = 0 = f_{yx}$

5pts

$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx} f_{yy} - f_{xy} f_{yx}$

$= (2)(-2) - 0 = -4$

Saddle @ ~~(0,0)~~ B $\boxed{(0,0,25)}$

203 E2

(7b) $f(x,y) = x^2 + 4y^2 - 50$

$f_x = 2x \qquad f_{xx} = 2$

$f_y = 8y \qquad f_{yy} = 8$

5 pts

$f_{xy} = 0 = f_{yx}$

$D = f_{xx} f_{yy} - f_{yx} f_{xy}$

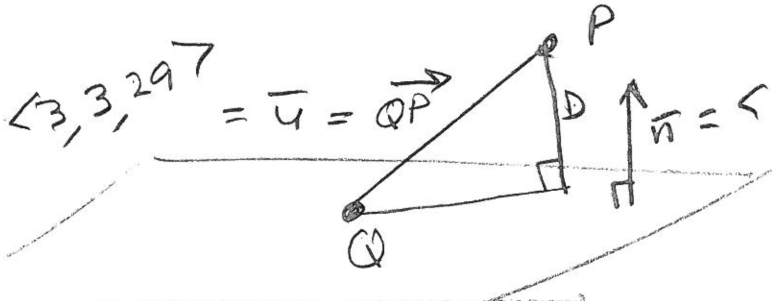
$= (2)(8) = 16$ Min @ $\boxed{(0,0,-50)}$

(8) P : $2x + 2y - z = 9$

$P(3,3,20) \qquad Q(0,0,-9)$ OR

$Q(0,1,-7)$, etc

$\langle 3,3,29\rangle = \bar{u} = \overrightarrow{QP} \qquad \bar{n} = \langle 2,2,-1\rangle$



$D = \dfrac{|\bar{n} \cdot \overrightarrow{QP}|}{|\bar{n}|} = \left|\dfrac{\bar{n} \cdot \bar{u}}{\bar{n}}\right|$

$= \left|\dfrac{\langle 2,2,-1\rangle \cdot \langle 3,3,29\rangle}{\sqrt{2^2+2^2+1^2}}\right| = \left|\dfrac{6+6-29}{\sqrt{9}}\right| = \boxed{\dfrac{17}{3}}$

5 pts

:203 E2

9

Minimize

$$\sqrt{(x-3)^2+(y-3)^2+(z-20)^2}$$

s.t.

$$z = 2x+2y-9$$

$\sqrt{\text{arg}}$ is increasing func of its arg., so we minimize

$$(x-3)^2+(y-3)^2+(2x+2y-9-20)^2 = g(x,y)$$

~~$g_x = 2(x-3)+2 = 2x-6+2 = 2x-4 \overset{\text{SET}}{=} 0 \Rightarrow \boxed{x=2}$~~

~~$g_y = 2(y-3)+$~~

$$g_x = 2(x-3)+2(2x+2y-29)(2)$$

$$= 2x-6+(4x+4y-58)(2)$$

$$= 2x-6+8x+8y-116$$

$$= 10x+8y-122 \overset{\text{SET}}{=} 0 \rightarrow$$

$$10x = -8y-122$$

$$\rightarrow x = -\frac{4}{5}y - \frac{61}{5}$$

203 E2

9 cnt'd

$g_y = 2(y-3) + 2(2x+2y-29)(2)$

$= 2y - 6 + 8x + 8y - 116$

$= 8x + 10y - 122$

$= 8\left(-\frac{4}{5}y - \frac{61}{5}\right) + 10y - 122$

$= \frac{-32y}{5} - \frac{488}{5} + \frac{50y}{5} - \frac{610}{5} \overset{SET}{=} 0 \rightarrow$

$\frac{18y}{5} - \frac{1098}{5} = 0$

$18y = 1098$

$\boxed{y = 61} \rightarrow$

$x = -\frac{4}{5}(61) - \frac{61}{5}$

$= \frac{-5(61)}{5} = \boxed{-61 = x}$

$3x + 2y - z = 9$

$-183 + 122 - 9 = z = -70$

$\boxed{(-61, 61, -70)}$

5 pts

$g_{xx} = 10 \qquad g_{yy} = 10$

$g_{xy} = 8 \qquad g_{yx} = 8$

$D = (10)(10) - (8)(8)$

$= 100 - 64 = 36 > 0 \rightarrow$ MIN

203 E2

B1 $f(\tau,\omega) = \int_{\sin\tau}^{\cos\omega} \pi(x^2 e^{\pi x})\,dx \longrightarrow$

$$f_\tau = \left(-\pi(\sin^2\tau\, e^{\pi\sin\tau})\right)(\cos\tau)$$

$$f_\omega = \pi(\cos^2\omega\, e^{\pi\cos\omega})(-\sin\omega)$$

B2 $f = y\int_0^x \xi^3 \arctan(\xi^4)\,d\xi$

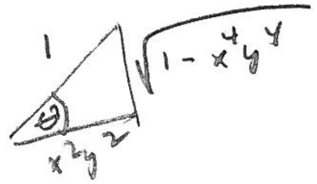
$$\Longrightarrow \boxed{f_x = y\, x^3 \arctan(x^4)}$$

$$f_y = \int_0^x \xi^3 \arctan(\xi^4)\,d\xi$$

B3 $f(x,y) = \arccos(x^2y^2)$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$f_x = \frac{2xy^2}{-\sin(\arccos(x^2y^2))} = \boxed{\frac{-2xy^2}{\sqrt{1-x^4y^4}}} = f_x$$



$$f_y = \frac{2x^2y}{-\sin(\arccos(x^2y^2))} = \boxed{\frac{-2x^2y}{\sqrt{1-x^4y^4}}} = f_y$$