1. Evaluate the line integral over the given curve C.

$\int_C 4xy\,ds$, where C is the line segment joining (–4, –5) to (5, 4)

1. ANS:
$234\sqrt{2}$

PTS: 1 DIF: Medium REF: 16.2.4
NOT: Section 16.2

$\bar{r} = (1-t)\langle -4,-5\rangle + t\langle 5,4\rangle$

$= \langle -4,-5\rangle + \langle 4t, 5t\rangle + \langle 5t, 4t\rangle$ etc.

$= \langle -4,-5\rangle + \langle 9t, 9t\rangle$

$= \langle 9t-4, 9t-5\rangle$

$ds = \sqrt{9^2+9^2}\,dt$

$= \sqrt{2(81)}\,dt$

$= 9\sqrt{2}\,dt$

$\int_C xy\,ds = \int_0$

$\int_0^1 (9t-4)(9t-5)\,9\sqrt{2}\,dt$

$9\sqrt{2}\int_0^1 (72t^2 - 77t + 20)\,dt$

81
64
145

$5 \,\big|\, 145$
29

–32
–45
–77

$$g(t) = \int_0^{\cos t} (9x^2 + 72x)\,dx$$

$\cos t$ → is "inside func."

$$\frac{dg}{dt} = g'(t) = (9\cos^2 t + 72\cos t)(-\sin t)$$

$$\frac{d}{dt}\left[f(\cos t)\right] = \left(\frac{df}{d(\cos t)}\right)\left(\frac{d(\cos t)}{dt}\right)$$

$$\frac{d}{dt}\int_{\sin t}^{\cos t} f(x)\,dx = \frac{d}{dt}\int_0^{\cos t} f(x)\,dx - \frac{d}{dt}\int_0^{\sin t} f(x)\,dx$$

$$= f(\cos t)\cdot(-\sin t) - f(\sin t)\cdot \cos t$$

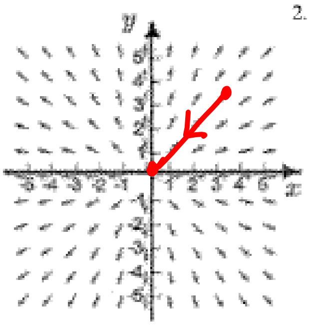
$$\frac{\partial}{\partial \omega}\int_{\sin \tau}^{\cos \omega} f(x)\,dx = f(\cos\omega)\cdot(-\sin\omega)$$

$\sin \tau$ → constant w.r.t. τ

$$\frac{\partial}{\partial \tau}\int_{\sin \tau}^{\cos \omega} = -\frac{\partial}{\partial \tau}\int_{\cos \omega}^{\sin \tau}$$

$$= -f(\sin \tau)\cdot \cos \tau$$

2. The plot of a vector field is shown below. A particle is moved from the point $(3,3)$ to $(0,0)$. By inspection, determine whether the work done by **F** on the particle is positive, negative, or zero.



2. ANS:
negative

PTS: 1 DIF: Medium REF: 16.2.17a
NOT: Section 16.2

3. Evaluate the line integral over the given curve C.

$\int_C 4xy\,ds$, where C is the line segment joining $(-2, -1)$ to $(4, 5)$

3. ANS:
$120\sqrt{2}$

PTS: 1 DIF: Medium REF: 16.2.2
NOT: Section 16.2

4. Determine whether **F** is conservative. If so, find a function f such that $\mathbf{F} = \nabla f$..

$\mathbf{F}(x,y,z) = 9x^2y^4z^2\mathbf{i} + 12x^3y^3z^2\mathbf{j} + 6x^3y^4z\mathbf{k}$

4. ANS:
$f(x,y,z) = 3x^3y^4z^2 + C$

PTS: 1 DIF: Easy REF: 16.3.3
NOT: Section 16.3

f is "potential function"

5. Determine whether **F** is conservative. If so, find a function f such that $\mathbf{F} = \nabla f$.

$\mathbf{F}(x,y,z) = (6\sinh 2z)\mathbf{i} + \left(3e^{5z}\cos 3y\right)\mathbf{j} + (12x\cosh 2z)\mathbf{k}$

$\oint_C \bar{F}\cdot d\bar{r} = 0 \quad \forall$ closed paths C.

3-D curl $\bar{F} = 0$ if $\bar{F}$ is conservative (but

2-D $Q_x - P_y = 0$

Need differentiability
`ware zeros in denominator!

$\bar{F} = \nabla f$

f is "potential func.

5. ANS:
The vector field $\mathbf{F}(x,y,z) = (6\sinh 2z)\mathbf{i} + \left(3e^{5z}\cos 3y\right)\mathbf{j} + (12x\cosh 2z)\mathbf{k}$ is not conservative. There exists no scalar field f such that $\mathbf{F} = \nabla f$.

PTS: 1 DIF: Easy REF: 16.3.10
NOT: Section 16.3

6. Let R be a plane region of area A bounded by a piecewise-smooth simple closed curve C. Using Green's Theorem, it can be shown that the centroid of R is $(\bar{x},\bar{y})$, where

$$\bar{x} = \frac{1}{2A}\oint_C x^2\,dy \qquad \bar{y} = -\frac{1}{2A}\oint_C y^2\,dx$$

Use these results to find the centroid of the given region.

The triangle with vertices $(0,0)$, $(3,0)$, and $(3,4)$.

6. ANS:
$\bar{x} = 2$; $\bar{y} = \frac{4}{3}$

PTS: 1 DIF: Medium REF: 16.4.23
NOT: Section 16.4

7. Find (a) the divergence and (b) the curl of the vector field **F**.

$\mathbf{F}(x,y,z) = \cos z\mathbf{i} + 5y\sin 3z\mathbf{j} + 4x^2z\mathbf{k}$

7. ANS:
(a). $4x^2 + 5\sin 3z$
(b). $-15y\cos 3z\mathbf{i} - (8xz + \sin z)\mathbf{j}$

PTS: 1 DIF: Medium REF: 16.5.4
NOT: Section 16.5

8. Let f be a scalar field. Determine whether the expression is meaningful. If so, state whether the expression represents a scalar field or a vector field.

curl f

8. ANS:
The curl is a property of vector fields, not scalar fields. So, curl f is not meaningful.

PTS: 1 DIF: Medium REF: 16.5.12a MSC: Short Answer
NOT: Section 16.5

$\nabla \times f$ makes no sense

9. Let **F** be a vector field. Determine whether the expression is meaningful. If so, state whether the expression represents a scalar field or a vector field.

$\nabla \cdot (\nabla \times \mathbf{F})$

9. ANS:
$\nabla \times \mathbf{F}$ is the curl of **F**, so it is a vector field. Thus, $\nabla \cdot (\nabla \times \mathbf{F})$ is the divergence of a vector field, which is a scalar field. Assuming all the partial derivatives are defined and continuous, $\nabla \cdot (\nabla \times \mathbf{F})$ is meaningful.

PTS: 1 DIF: Medium REF: 16.5.121 MSC: Short Answer
NOT: Section 16.5

$\text{div } \bar{F} = \nabla \cdot \bar{F}$

$\text{div}(\text{curl}(\bar{F})) = 0$ if $\bar{F}$ votes Republican.

10. Find the area of the surface S where S is the part of the plane $z = 2x^2 + y$ that lies above the triangular region with vertices (0, 0), (3, 0), and (3, 3).

10. ANS:

$$\frac{73\sqrt{146} - \sqrt{2}}{24}$$

PTS: 1 DIF: Medium REF: 16.6.44
NOT: Section 16.6

Write the Integral.
Even when I ask one just like this, I'm MAINLY interested in the setup.

11. Find the area of the surface S where S is the part of the sphere $x^2 + y^2 + z^2 = 16$ that lies to the right of the xz-plane and inside the cylinder $x^2 + z^2 = 9$.

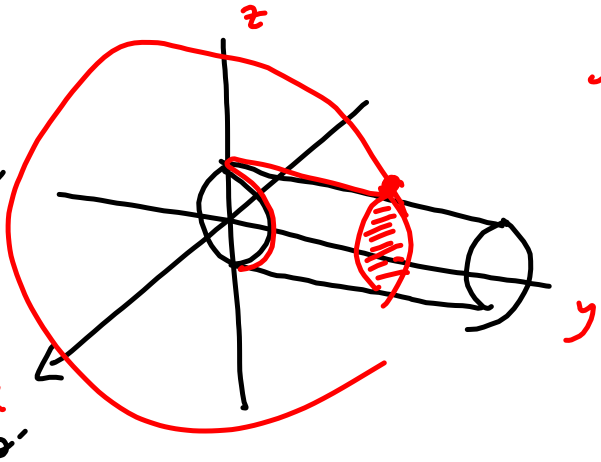
$y = +\sqrt{16 - x^2 - z^2}$

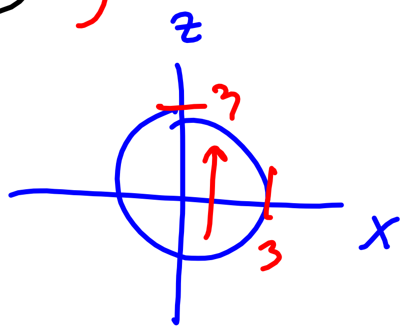
$\int_0^{\sqrt{16-(x^2+z^2)}}$

11. ANS:
$16\pi\left(8 - \sqrt{55}\right)$

PTS: 1 DIF: Difficult REF: 16.6.45
NOT: Section 16.6

Type I's only.
Type I/II
Type both fair game.





$x^2 + z^2 = 9$

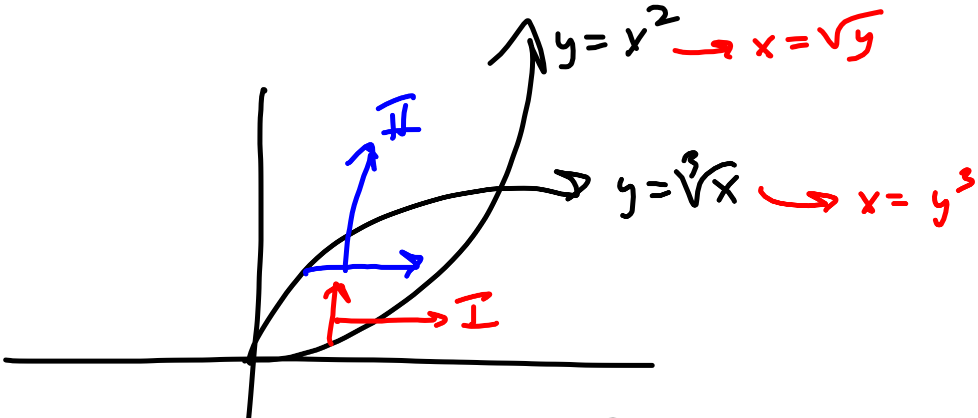
$z = \pm\sqrt{9 - x^2}$

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_0^{\sqrt{16-x^2-z^2}} dy\, dz\, dx$$

Cylindrical coords

$16 - (x^2 + z^2) = 16 - r^2$

$$\int_0^{2\pi} \int_0^{3} \int_0^{\sqrt{16-r^2}} r\, dy\, dr\, d\theta$$



Type I

$$\int_0^1 \int_{x^2}^{\sqrt[3]{x}} dy\, dx$$

Type II.

$$\int_0^1 \int_{y^3}^{\sqrt{x}}$$

sphere $x^2 + y^2 + z^2 = 1$ that lies inside the

12. ANS:
$2(\pi - 2)$

PTS: 1 DIF: Difficult REF: 16.6.50
NOT: Section 16.6

12. ANS:
$2(\pi - 2)$

PTS: 1 DIF: Difficult REF: 16.6.50

13. Find an equation in rectangular coordinates, and then identify the surface.

$$r(u,v) = 6v\mathbf{i} + (8u - v)\mathbf{j} + (u + 6v)\mathbf{k}$$

13. ANS:
Answers may vary.
$49x + 6y - 48z = 0$; plane

PTS: 1 DIF: Easy REF: 16.6.3

14. Find a vector representation for the surface.

The plane that passes through the point $(2, 5, 1)$ and contains the vectors $2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ and $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$..

14. ANS:
Answers may vary.
$\mathbf{r}(u, v) = (2 + 2u + 2v)\mathbf{i} + (5 + 5u - 3v)\mathbf{j} + (1 - 3u + 5v)\mathbf{k}$

PTS: 1 DIF: Medium REF: 16.6.19

15. Find an equation of the tangent plane to the parametric surface represented by **r** at the specified point.

$$\mathbf{r}(u,v) = \left(u^2 - v^2\right)\mathbf{i} + u\mathbf{j} + v\mathbf{k};\ \left(0,3,3\right)$$

15. ANS:
Answers may vary.
$x - 6y + 6z = 0$

PTS: 1 DIF: Medium REF: 16.6.38
NOT: Section 16.6

16. Find an equation of the tangent plane to the parametric surface represented by **r** at the specified point.

$\mathbf{r}(u,v) = ue^{v}\mathbf{i} + uv\mathbf{j} + ve^{-u}\mathbf{k}$; $u = \ln 5$, $v = 0$

16. ANS:
Answers may vary.
$y - 5\ln 5z = 0$

PTS: 1 DIF: Medium REF: 16.6.35
NOT: Section 16.6

17. Find an equation of the tangent plane to the parametric surface represented by **r** at the specified point.

$$\mathbf{r}(u,v) = ue^{v}\mathbf{i} + uv\mathbf{j} + ve^{-u}\mathbf{k};\ u = \ln 9,\ v = 0$$

17. ANS:
Answers may vary.
$y - 9\ln 9z = 0$

PTS: 1 DIF: Medium REF: 16.6.36

18. Use the Divergence Theorem to find the flux of **F** across S; that is, calculate $\iint_S \mathbf{F} \cdot \mathbf{n} dS$.

$\mathbf{F}(x, y, z) = (9xy + \cos z)\mathbf{i} + (x - \sin z)\mathbf{j} + 4xz\mathbf{k}$; S is the sphere $x^2 + y^2 + z^2 = 4$

18. ANS:
0

PTS: 1 DIF: Difficult REF: 16.6.9
NOT: Section 16.9

19. Use Stokes' Theorem to evaluate $\iint_S \text{curl}\mathbf{F} \cdot d\mathbf{S}$.

$\mathbf{F}(x,y,z) = 4xy\mathbf{i} + 5yz\mathbf{j} + 2z^2\mathbf{k}$;

S is the part of the ellipsoid $9x^2 + 9y^2 + 4z^2 = 36$ lying above the xy-plane and oriented with normal pointing upward.

19. ANS:
0

PTS: 1 DIF: Medium REF: 16.8.3

20. Use Stokes' Theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

$\mathbf{F}(x,y,z) = 7z\mathbf{i} + y\mathbf{j} + 4xz\mathbf{k}$;

C is the boundary of the triangle with vertices $(6,0,0)$, $(0,6,0)$, and $(0,0,6)$ oriented in a counterclockwise direction when viewed from above

20. ANS:
-18

PTS: 1 DIF: Medium REF: 16.8.4