§16.9 #9 Compute div $\bar{F}$ before you switch to your brown britches.

"Fat sphere?!"

$x^8 + y^8 + z^8 = 8$

Source -vs- sink

div $\bar{F}$ measures the tendency (in fluid flow) to diverge at (x, y, z)

Divergence Theorem in words:

The flux of $\bar{F}$ across S (closed surface.) is the triple integral of the divergence of $\bar{F}$ over the solid bounded by S

$$\iint_S \bar{F} \cdot d\bar{S} = \iiint_E \operatorname{div} \bar{F} \, dV$$

§16.9

① Verify D.T. for

$\bar{F} = \langle 3x, xy, 2xz \rangle$ over

S: cube bdd by $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, $z=1$

$S_1 \quad S_2 \quad S_3 \quad S_4 \quad S_5 \quad S_6$

Do $\iint$ 1st

$$\iint_S = \sum_{k=1}^{6} \iint_{S_k}$$

$\nearrow \bar{n}$

dydz

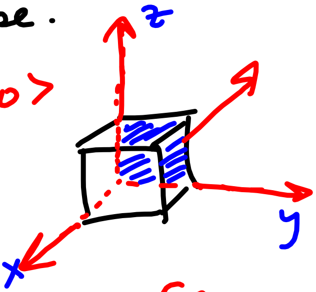
S_1: $\bar{F}\cdot d\bar{S} = \bar{F}\cdot \dfrac{\bar{r}_y \times \bar{r}_z}{\|\bar{r}_y \times \bar{r}_z\|} dS = \bar{F}\cdot \dfrac{\bar{r}_y \times \bar{r}_z}{\cancel{\|\bar{r}_y \times \bar{r}_z\|}} \cancel{\|\bar{r}_y \times \bar{r}_z\|} dA$

Don't need either one 'cause $\bar{n}$ is so easy!

But $\bar{n}$ is EZ.

for sides of a cube. $= \bar{F}\cdot(\bar{r}_y \times \bar{r}_z)\, dy\, dz$

$\bar{n} = \langle -1, 0, 0 \rangle$



$$\iint_{S_1} \bar{F}\cdot d\bar{S} = \iint_{S_1} \langle 3x, xy, 2xz\rangle \cdot \langle -1,0,0\rangle\, dy\, dz$$

$$= \int_0^1\int_0^1 -3x\, dy\, dz = \iint_{S_1} 0 = \boxed{0} \quad S_1$$

S_2 $x = 1$:

$F = \langle 1, y, z \rangle$

$\bar{r}_y = \langle 0, 1, 0 \rangle$

$\times \quad \bar{r}_z = \langle 0, 0, 1 \rangle$

$\langle 1, 0, 0 \rangle$

Idiot: do by inspection.

(Note to self.)

$\bar{F}\cdot\bar{n} = 3(1) = 3$

$\langle P, Q, R\rangle \cdot \langle 1,0,0\rangle = P = 3x$, but $x \equiv 1$

$$\int_0^1\int_0^1 3\, dy\, dz$$

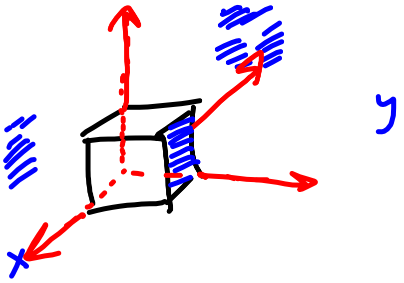
$$= \int_0^1 [3y]_0^1 dz = \int_0^1 3\, dz = \boxed{3} \quad S_2$$

$S_3: y=0$

$\bar{n} = \langle 0,-1,0\rangle$

$\bar{F}\cdot\bar{n} = \langle 3x, xy, 2xz\rangle \cdot \langle 0,-1,0\rangle = -xy = 0$

$S_3: \boxed{0}$



$S_4: y=1$

$\bar{n} = \langle 0,1,0\rangle$

$\bar{F}\cdot\bar{n} = xy = x$

$$\int_0^1\int_0^1 x\,dz\,dx = \int_0^1 [xz]_{z=0}^{z=1} = \int_0^1 x\,dx = \tfrac{1}{2}x^2\Big]_0^1 = \boxed{\tfrac{1}{2}} \quad S_4$$

$S_5: z=0$

$\bar{n} = \langle 0,0,-1\rangle$

$$\bar{F}\cdot\bar{n} = \langle 3x, xy, 2xz\rangle\cdot\langle 0,0,-1\rangle = -2xz = \boxed{0} \quad S_5$$

$S_6: z=1$

$\bar{n} = \langle 0,0,1\rangle$

$\bar{F}\cdot\bar{n} = 2xz = 2x$

$$\int_0^1\int_0^1 2x\,dy\,dx = \int_0^1 [2xy]_0^1\,dx = \int_0^1 2x\,dx = x^2\Big]_0^1 = \boxed{1} \quad S_6$$

$$\iint_S \bar{F}\cdot d\bar{S} = \sum_{k=1}^{6}\int_{S_k}\bar{F}\cdot d\bar{S} = 0+3+0+\tfrac{1}{2}+0+1 = \boxed{\tfrac{9}{2}}$$

Other way:

$$\iiint_E \operatorname{div} \bar{F}\, dV$$

$\operatorname{div} \bar{F} =$

$$\nabla \cdot \langle 3x, xy, 2xz \rangle = 3 + x + 2x = 3x+3 = 3(x+1)$$

$$3\int_0^1\int_0^1\int_0^1 (x+1)\,dy\,dz\,dx = 3\left[\int_0^1\int_0^1 (x+1)\,dz\,dx\right]$$

$$= 3\int_0^1 (x+1)\,dx = 3\left[\tfrac{1}{2}x^2 + x\right]_0^1$$

$$= 3\left[\tfrac{1}{2}+1\right] = 3\left[\tfrac{3}{2}\right] = \boxed{\tfrac{9}{2}}$$

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$\bar{r} = \langle x, y, z \rangle$

$$\bar{F}(\bar{r}) = \frac{c\bar{r}}{\|\bar{r}\|^3}$$

Show that

$\iint_S \bar{F} \cdot d\bar{S}$ is independent of any S containing the origin.

S_1 = outer sphere

S_2 = inner sphere

$$\iint_{S_1} - \iint_{S_2} = 0.$$

Mimics thinking in proof of Green's.