Special Case $z=g(x,y)$ & x & y are parameters...

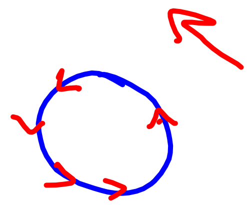
It's easier to re-derive curl $\bar{F}$ via $\nabla \times \bar{F}_y$ than to memorize $\langle -g_x, -g_y, 1\rangle$

§16.8

$$\begin{array}{r} \langle \partial/\partial x, \partial/\partial y, \partial/\partial z\rangle \\ \times\ \langle x, y, g(x,y)\rangle \\ \hline \langle -g_x, -g_y, 1\rangle \end{array}$$

$\boxed{E3}$ $\int_C \bar{F}\cdot d\bar{r}$ Line Integral of tangential component of $\bar{F}$.

$= \iint_S \text{curl}\,\bar{F}\cdot d\bar{S}$ surface integral of NORMAL component of curl $\bar{F}$



$\boxed{d\bar{S} = \bar{n}\, dS} = \bar{n}\ \|\bar{r}_u\times\bar{r}_v\|\, dA$

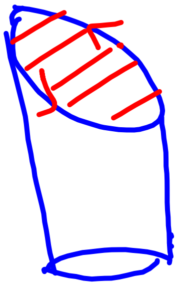
$= \dfrac{\bar{r}_u\times\bar{r}_v}{\|\bar{r}_u\times\bar{r}_v\|}\ \|\bar{r}_u\times\bar{r}_v\|\, dA$

$= (\bar{r}_u\times\bar{r}_v)\, dA$

$\iint_S \text{curl}\,\bar{F}\cdot\bar{n}\, dS$

$\boxed{E3}$ C is ellipse atop $x^2+y^2=1$, where $y+z=2$ intersects it $\qquad \vec{F}=\langle -y^2, x, z^2\rangle$

want $\displaystyle\int_C \vec{F}\cdot d\vec{r} =$



$$\int_C \langle -y^2, x, z^2\rangle \cdot$$

Cylindrical coords:

$x = r\cos\theta = \cos\theta$

$y = r\sin\theta = \sin\theta$

$z = z = 2-\sin\theta$

$y+z=2$

$z=2-y$

$$\vec{r} = \langle x, y, 2-y\rangle$$
$$= \langle \cos\theta, \sin\theta, 2-\sin\theta\rangle$$
$$\vec{r}\,'(\theta) = \langle -\sin\theta, \cos\theta, -\cos\theta\rangle$$
$$d\vec{r} = \vec{r}\,'(\theta)\,d\theta$$

& so $\vec{F}\cdot d\vec{r} = \langle -y^2, x, z^2\rangle \cdot \langle -\sin\theta, \cos\theta, -\cos\theta\rangle$

$$= \int_C \langle -\sin^2\theta, \cos\theta, \sin^2\theta - 4\sin\theta + 4\rangle \cdot \langle -\sin\theta, \cos\theta, -\cos\theta\rangle\, d\theta$$

$=$

$$= \int_0^{2\pi} \left(\sin^3\theta + \cos^2\theta - \cos\theta\sin^2\theta + 4\sin\theta\cos\theta - 4\cos\theta\right) d\theta$$

$= \pi$

Doing it w/o STOKES is painful.

$\bar{F} = \langle -y^2, x, z^2 \rangle$

$\text{curl}\,\bar{F} = \langle 0, 0, 2y+1 \rangle$

$$\iint_S \text{curl}\,\bar{F} \cdot d\bar{S}$$

$\bar{r} = \langle x, y, 2-y \rangle$

$\bar{r}_x \quad \langle 1, 0, 0 \rangle$

$\bar{r}_y \quad \langle 0, 1, -1 \rangle$

$\langle 0, 1, 1 \rangle$

$$= \left(d\bar{S} = (\bar{r}_x \times \bar{r}_y)\, dA \right.$$

$$\left. \text{curl}\,\bar{F} \cdot d\bar{S} = \langle 0, 0, 2y+1 \rangle \cdot \langle 0, 1, 1 \rangle = 2y+1 \right)$$

$$= \iint_D (2y+1)\, dA = \int_0^{2\pi}\int_0^1 (2r\sin\theta + 1)\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \left[\frac{2}{3} r^3 \sin\theta + \frac{r^2}{2} \right]_0^1 d\theta$$

$$= \int_0^{2\pi} \left[\frac{2}{3}\sin\theta + \frac{1}{2} \right] d\theta =$$

$$\left[0 \right] + \left. \frac{1}{2}\theta \right]_0^{2\pi} = \pi$$

§16.8 #s 1-3, 7, 8, 10, 11, 16, 17 (Read #20)

#11 I'll getcha the pics.

§16.9 #s 1, 5, 7, 12, 19, 20

Divergence Theorem

Put this into words.

$$\iint_S \bar{F} \cdot d\bar{S} = \iiint_E \operatorname{div} \bar{F} \, dV$$

$$\operatorname{div} \bar{F} = \nabla \cdot \bar{F} = P_x + Q_y + R_z$$

$$\operatorname{curl} \bar{F} = \nabla \times \bar{F}$$