§16.7

Surfaces orientation

Special case $z = g(x,y)$

Need an $\bar{n}$ = normal

$\bar{n} = (\bar{r}_x \times \bar{r}_y)\left(\frac{1}{\|\bar{r}_x \times \bar{r}_y\|}\right)$

$\bar{r}$ = surface S

$\bar{r} = \langle x, y, g(x,y) \rangle$

$$\begin{array}{rl} & \bar{r}_x = \langle 1, 0, g_x \rangle \\ \times & \bar{r}_y = \langle 0, 1, g_y \rangle \\ \hline \end{array}$$

$\langle -g_x, -(g_y), 1 \rangle = \bar{r}_x \times \bar{r}_y$

$\|\bar{r}_x \times \bar{r}_y\| = \sqrt{g_x^2 + g_y^2 + 1}$

In this case, $\bar{n}$ = unit normal $= \dfrac{\langle -g_x, -g_y, 1 \rangle}{\sqrt{g_x^2 + g_y^2 + 1}}$

$$= \frac{\langle -\frac{\partial g}{\partial x}, -\frac{\partial g}{\partial y}, 1 \rangle}{\sqrt{\left(\frac{\partial g}{\partial x}\right)^2 + \left(\frac{\partial g}{\partial y}\right)^2 + 1}}$$

Surface Integrals of Vector Fields

Flux (Flow) across the boundary

Fluid flow ρ = density

$\bar{v}$ = velocity

$\bar{n}$ = Normal to the surface

Then flow thru (across) a small patch:

$$(\rho \bar{v} \cdot \bar{n}) A(S_{ij})$$

Add 'em up $\sum\sum \longrightarrow$

$$\iint_S \rho \bar{v} \cdot \bar{n} \, dS$$

$$= \iint_S \rho(x,y,z) \, \bar{v}(x,y,z) \cdot \bar{n}(x,y,z) \, dS$$

= Flow Rate Across the surface S.

= Mass/unit time This is ...

$\boxed{\text{Flux Integral}}$

Now: $\bar{F}\cdot(\bar{r}_x \times \bar{r}_y)$ in the case

$z=g(x,y)$, $\bar{F}=\langle P,Q,R\rangle$

So $\bar{F}\cdot(\bar{r}_x\times\bar{r}_y)$ $\qquad \langle -g_x,-g_y,1\rangle=\bar{r}_x\times\bar{r}_y$

$$=\langle P,Q,R\rangle\cdot\langle -g_x,-g_x,1\rangle$$

$$= -Pg_x-Qg_y+R$$

$d\bar{S}=$

So, $\displaystyle\iint_S \bar{F}\cdot d\bar{S}$

$\bar{S}$ is $\bar{r}(t)$

~~$d\bar{S}=\bar{r}'(t)dt$~~

No.

$$=\iint_D(-Pg_x-Qg_y+R)\,dA$$

$\boxed{10}$

$\bar{E}$ = electric Field

$$\iint_{S'} \bar{E} \cdot d\bar{S} = \text{electrix flux}$$

Gauss's Th net charge enclosed by closed surface S' is

$\boxed{11}$ $$Q = \varepsilon_0 \iint_{S'} \bar{E} \cdot d\bar{S}$$

$$\varepsilon_0 \approx 8.9 \times 10^{-12} \frac{C^2}{N\text{-}m^2}$$

Heat flow

$u(x,y,z)$ = temp

$\Rightarrow \bar{F} = -K \nabla u$

K = heat conductivity

Rate of flow across S is

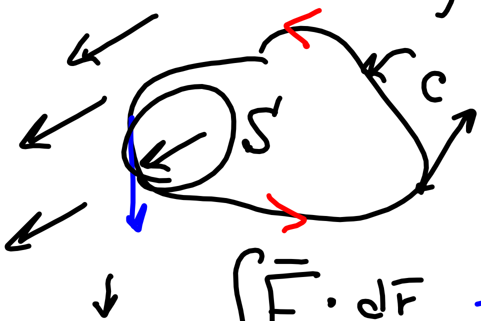
$$\iint_S \bar{F} \cdot d\bar{S} = -\iint_S K \nabla u \cdot d\bar{S}$$

#24 : $\bar{n}$ is downward $\langle 0,0,-1\rangle = \bar{n}$

§16.8 Stokes' Thm.

$\boxed{\mathrm{I}}$ Everything is smooth & oriented.

C = boundary of S



$\bar{n}$ out of the page

$d\bar{r} = \bar{r}'(t)\,dt$

$$\int_C \bar{F} \cdot d\bar{r} = \iint_S \operatorname{curl} \bar{F} \cdot d\bar{S}$$

Green's Theorem

$$\int_C P\,dx + Q\,dy = \iint_D (Q_x - P_y)\,dA$$

STOKES' over the weekend:

Copy & understand the proof. Add details as needed.

Some tools you'll need:

§14.5 Chain Rule.

If Q, R are functions of x, y, z,

what's $\frac{\partial}{\partial x}(Q) = \frac{\partial Q}{\partial x}\cdot\frac{\partial x}{\partial x} + \frac{\partial Q}{\partial x}\cdot\frac{\partial y}{\partial x} + \frac{\partial Q}{\partial x}\cdot\frac{\partial z}{\partial x}$

$z = g(x,y)$ & y is independent of x

we have

$$\frac{\partial Q}{\partial x} + 0 + \frac{\partial Q}{\partial x}\cdot\frac{\partial g}{\partial x}$$