12–18 (a) Find a function f such that $\mathbf{F} = \nabla f$ and (b) use part (a) to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve C.

12. $\mathbf{F}(x, y) = x^2\,\mathbf{i} + y^2\,\mathbf{j}$,
C is the arc of the parabola $y = 2x^2$ from $(-1, 2)$ to $(2, 8)$

If $\bar{F} = \langle P, Q \rangle = \langle x^2, y^2 \rangle = \nabla f$ for some f, $= \langle f_x, f_y \rangle$

then $f_x = x^2$

$\Rightarrow f = \frac{1}{3}x^3 + \alpha(y)$

$\Rightarrow f_y = 0 + \frac{\partial}{\partial y}(\alpha(y))$

$= \alpha'(y) = y^2$

Variation of Parameters

$\Rightarrow \alpha(y) = \frac{1}{3}y^3 + K$

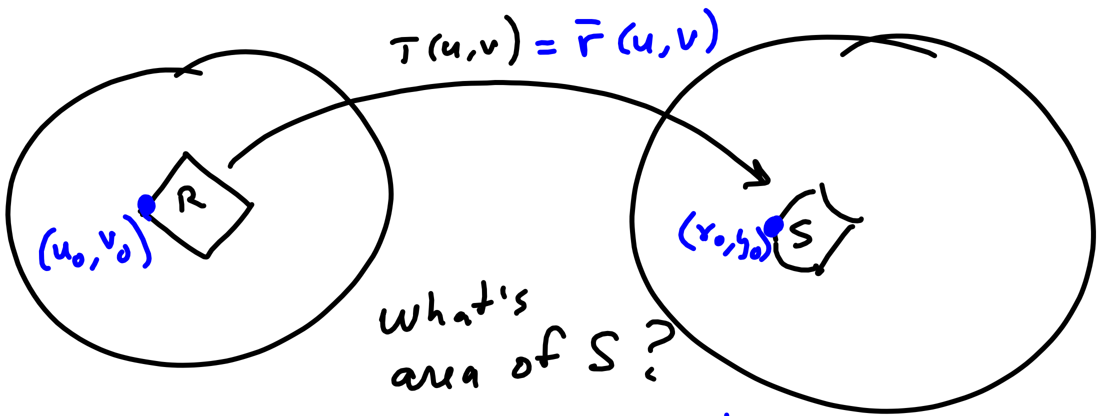
$\Rightarrow \boxed{f = \frac{1}{3}x^3 + \frac{1}{3}y^3 + K}$

In practice, we take $K = 0$.

How do we KNOW that f existed?
" " " " $\bar{F} = \nabla f$ for some f?

$Q_x = P_y$ & everything in sight is SMOOTH, baby.

MONDAY Bring your 16.1
We'll do the 3-D vector fields.

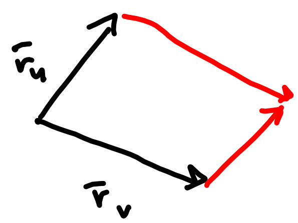


what's area of S?

We approximate w/ tangent plane.

$\bar{n} = \bar{r}_u \times \bar{r}_v$ is normal to the tangent plane

This is all just §15.10.

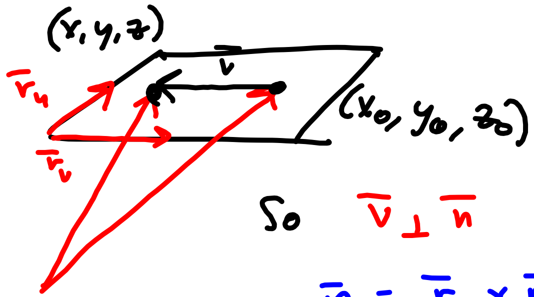


Area is $\bar{r}_u \times \bar{r}_v$

Let (x_0, y_0, z_0) be a point in the tangent plane.

Let (x, y, z) be another point.

Then $\bar{v} = \langle x - x_0, y - y_0, z - z_0 \rangle$ is a vector in the plane.



So $\bar{v} \perp \bar{n}$

$\bar{n} = \bar{r}_u \times \bar{r}_v$

$$\bar{n} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} = \langle n_1, n_2, n_3 \rangle$$

$$\bar{n} \cdot \bar{v} = 0$$

$$\bar{n} \cdot \bar{v} = n_1 (x - x_0) + n_2 (y - y_0) + n_3 (z - z_0) = 0$$

$\boxed{D6}$ $A(S) =$ Area of Surface

$S = \{ \bar{r}(u, v) \mid (u, v) \in D \}$ is

$$A(S) = \iint_D |\bar{r}_u \times \bar{r}_v| \, du\, dv$$

Special Case, $z = f(x,y)$

$\Rightarrow \bar{r} = \langle x, y, f(x,y) \rangle$

$$\begin{array}{rl} & \bar{r}_x = \langle 1, 0, f_x \rangle \\ \times & \bar{r}_y = \langle 0, 1, f_y \rangle \\ \hline \end{array}$$

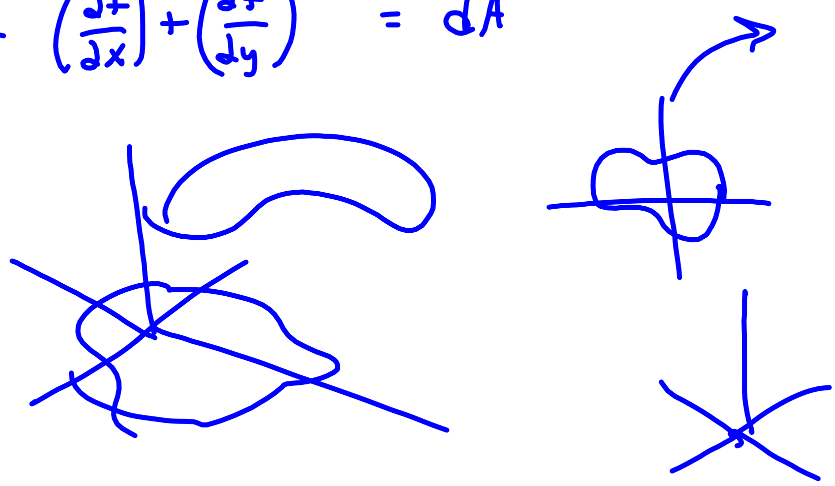
$\bar{r}_u \times \bar{r}_v = \langle -f_x, -f_y, 1 \rangle$, so

$$|\bar{r}_u \times \bar{r}_v| = \sqrt{f_x^2 + f_y^2 + 1}$$

Should look familiar!

Book gave us

$$\sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} = dA$$



So for the special case $z = f(x,y)$,

we get $\boxed{9}$ $$A(S) = \iint_D \underbrace{\sqrt{1 + f_x^2 + f_y^2}}_{|\bar{r}_u \times \bar{r}_v|}\, dA$$

Recall Jacobian

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_v & y_u \end{vmatrix} = \begin{vmatrix} x_u & y_u \\ x_v & y_v \end{vmatrix}$$

$\det(A^T)$

$= \det(A)$

$$= \bar{r}_u \times \bar{r}_v$$

$$\bar{r} = \langle x, y \rangle$$

$$\bar{r}_u = \langle x_u, y_u \rangle$$

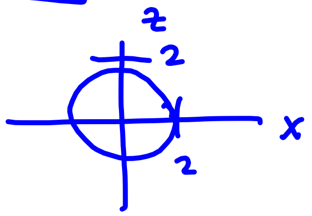
$$\bar{r}_v = \langle x_v, y_v \rangle$$

§16.6 #s 3, 4, 7, 10, 20, 21, 30, 33, 36, 41–44, 48

or 41, 48?

Some Examples. $\langle 2\cos t, t, 2\sin t \rangle$

$\boxed{E1}$ $\vec{r} = \langle 2\cos u, v, 2\sin u \rangle$



$= \langle 2\cos u, y, 2\sin u$

Cylinder, with axis = y-axis

xz:

$(2\cos t)^2 + (2\sin t)^2$

$= 4(\cos^2 t + \sin^2 t) = 4$, so $x^2 + z^2 = 4$

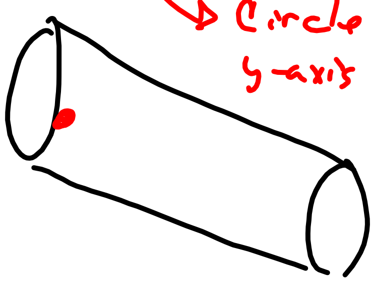
Grid Curves: Hold u constant

Hold v constant.

v = constant : Circle about y-axis.

u = constant : Line parallel to y-axis.

Circle centered about the y-axis in the plane $y = V = \text{constant}$



$\bar{r} = \langle (2+\sin v)\cos u, (2+\sin v)\sin u, u+\cos v \rangle$

$u = \text{const}$

$\langle c_1(2+\sin v), c_2(2+\sin v), u_0+\cos v \rangle$

Tough to visualize.

Remind me if we can look at these space curves

I suck!