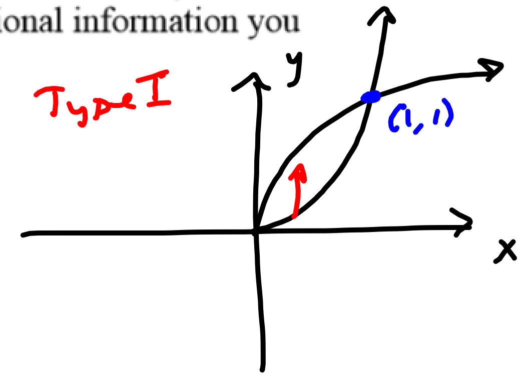
1. (15 pts) Evaluate the iterated integral $\int_0^1 \int_{\sqrt{3x}}^{e^{x^2}} 8xy\,dy\,dx$. A sketch of the Type I region $\boldsymbol{R}$ over which this integral is taken is given on the right, with some additional information you might find helpful for #2.

$$\int_0^1\int_{x^2}^{\sqrt{x}} 8xy\,dy\,dx$$

Type I



$$= \int_0^1 \left[8x\frac{y^2}{2}\right]_{x^2=y}^{\sqrt{x}=y} dx$$

$$= \int_0^1 \left[4x^2 - 4x^5\right] dx = \left[\frac{4}{3}x^3 - \frac{2}{3}x^6\right]_0^1 = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$$

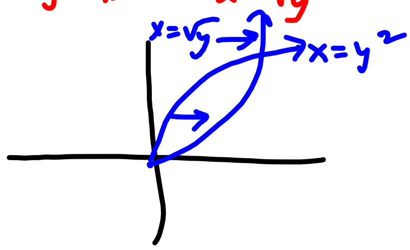
$8x\frac{(\sqrt{x})^2}{2}$ - $8x\frac{(x^2)^2}{2}$

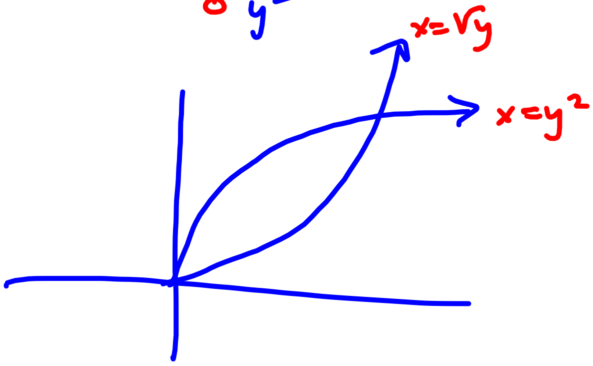
$y = \sqrt{x} \Rightarrow x = y^2$

$y = x^2 \Rightarrow x = \sqrt{y}$

Type II version

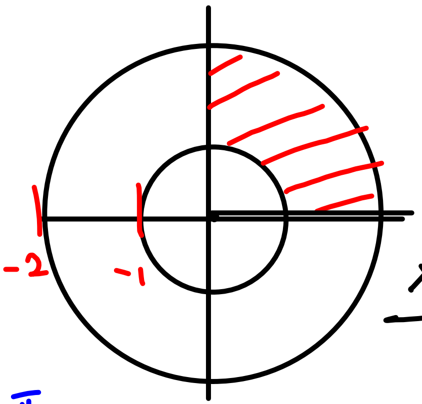
$$\int_0^1\int_{y^2}^{\sqrt{y}} 8xy\,dx\,dy$$





3. (15 pts) $\boxed{\text{Evaluate}}$ the iterated integral $\iint_D (x+y)\,dA$, where D is the region in the 1st quadrant, between the circles $x^2+y^2=4$ and $x^2+y^2=1$, by converting to polar coordinates.

write



$x = r\cos\theta$

$y = r\sin\theta$

$\bar{r} = \langle r\cos\theta, r\sin\theta, 0\rangle$

$\bar{r}_r = \langle \cos\theta, \sin\theta, 0\rangle$

$\times\ \bar{r}_\theta = \langle -r\sin\theta, r\cos\theta, 0\rangle$

$\langle 0, 0, r\cos^2\theta + r\sin^2\theta\rangle$

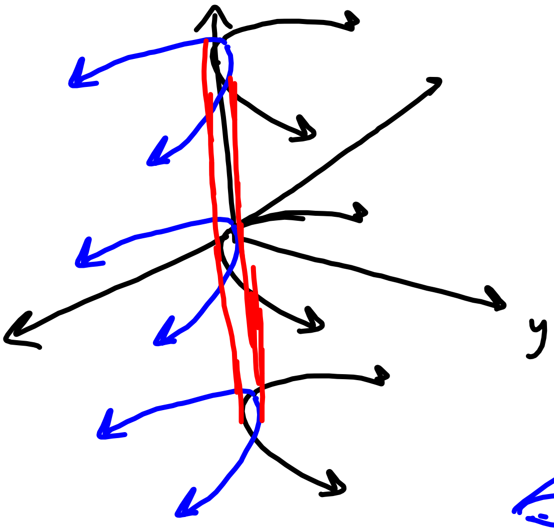
$= \langle 0, 0, r\rangle$

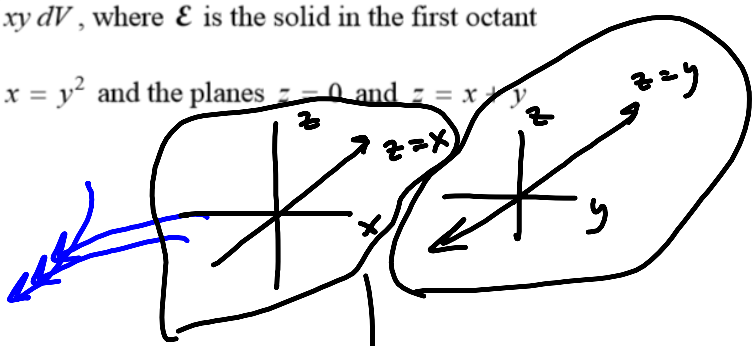
The jacobian!

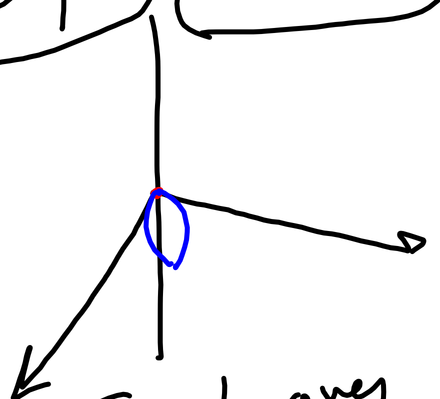
$$\int_0^{\frac{\pi}{2}}\int_1^2 (r\cos\theta + r\sin\theta)\,r\,dr\,d\theta$$

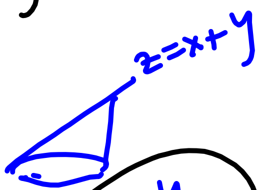
So, $dA = r\,dr\,d\theta$

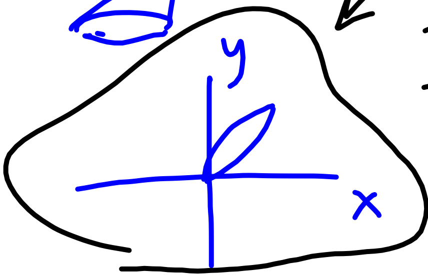
4. (15 pts) Evaluate the triple integral $\iiint_{\mathcal{E}} xy\, dV$, where $\mathcal{E}$ is the solid in the first octant bounded by the parabolic cylinders $y = x^2, x = y^2$ and the planes $z = 0$ and $z = x + y$.











Type 1 over
Type I

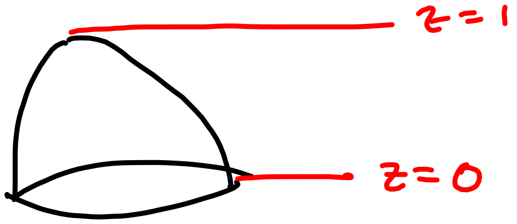
$$\int_0^1 \int_{x^2}^{\sqrt{x}} \int_0^{x+y} xy\, dz\, dy\, dx$$

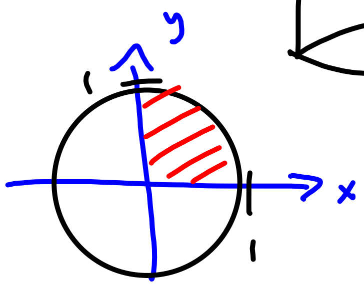
$$\int_0^1 \int_{y^2}^{\sqrt{y}} \int_0^{x+y} xy\, dz\, dx\, dy$$

Type 1 over
Type II

5. (15 pts) Evaluate the triple integral $\iiint_{\mathcal{E}} \left(x^3 + xy^2\right) dV$, where $\mathcal{E}$ is the solid in the first octant that lies beneath the paraboloid $z = 1 - x^2 - y^2$. Hint: Converting to Cylindrical Coordinates after you've found the triple integrals limits of integration will make evaluation easier.

→ write





$z=0:$ $1 - x^2 - y^2 = 0$

$x^2 + y^2 = 1 \Longrightarrow y = \pm\sqrt{1-x^2}$

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{1-(x^2+y^2)} x\left(x^2+y^2\right) dz\,dy\,dx$$

$$\sqrt{1-x^2}$$
$$= \sqrt{1 - r^2\cos^2\theta}$$
$$= \sqrt{r^2\sin^2\theta}$$
$$= |r\sin\theta|$$
$= r\sin\theta$, since $r>0$, $\sin\theta \geq 0$ in 1^{st} quad.

$$\int_0^1 \int_0^{\frac{\pi}{2}} \int_0^{1-r^2} (r\cos\theta) r^2 \cdot r \, dz\,d\theta\,dr$$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 \int_0^{1-r^2} r^4 \cos\theta \, dz\,dr\,d\theta$$ is probably easier to crank out

6. (15 pts) Compute the Jacobian for the transformation $u = x + y, v = 2x + 3y$.

$u = x+y \Rightarrow x = u-y$

$v = 2x+3y \Rightarrow v = 2(u-y)+3y$

$= 2u-2y+3y$

$= 2u+y = v$

$\Rightarrow \boxed{y = v-2u}$

$x = u-y$

$x = u-(v-2u)$

$= u-v+2u$

$\boxed{x = 3u-v}$

$\bar{r} = \langle x, y, 0 \rangle$

$= \langle 3u-v, -2u+v, 0 \rangle$

$\bar{r}_u = \langle 3, -2, 0 \rangle$

$\times \quad \bar{r}_v = \langle -1, 1, 0 \rangle$

$\langle 0, 0, 1 \rangle$ The Jacobean

$$\frac{\partial(x,y)}{\partial(u,v)} = 1$$

Make sure you can do the 3-D version, $\therefore \int$

$x = g(u,v,w) = x(u,v,w)$

$y = h(u,v,w)$ parts are given

$z = k(u,v,w)$

$\bar{r} = \langle g, h, k \rangle = \langle x, y, z \rangle$

$\bar{r}_u = \langle g_u, h_u, k_u \rangle$

$\bar{r}_v = \langle g_v, h_v, k_v \rangle$

$\bar{r}_w = \langle g_w, h_w, k_w \rangle$

2-D: $\begin{vmatrix} i & j & k \\ g_u & h_u & 0 \\ g_v & h_v & 0 \end{vmatrix}$

$$\text{Jacobean} = \begin{vmatrix} g_u & h_u & k_u \\ g_v & h_v & k_v \\ g_w & h_w & k_w \end{vmatrix}$$

Book: $\begin{vmatrix} g_u & g_v & g_w \\ h_u & h_v & h_w \\ k_u & k_v & k_w \end{vmatrix} =$

$\bar{r}_u \cdot (\bar{r}_v \times \bar{r}_w)$ Vol of parallelepiped.

Another Approach:

$\bar{r}_v \times \bar{r}_w$

$\langle g_v, h_v, k_v \rangle$

$\times \langle g_w, h_w, k_w \rangle$

$\langle \qquad \rangle$

$\bar{r}_u \cdot \langle \qquad \rangle$ = Scalar triple product.