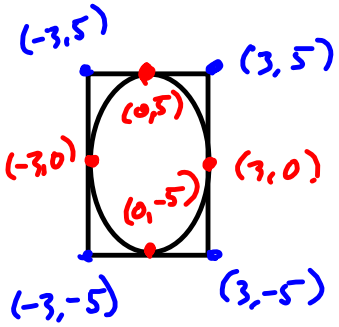
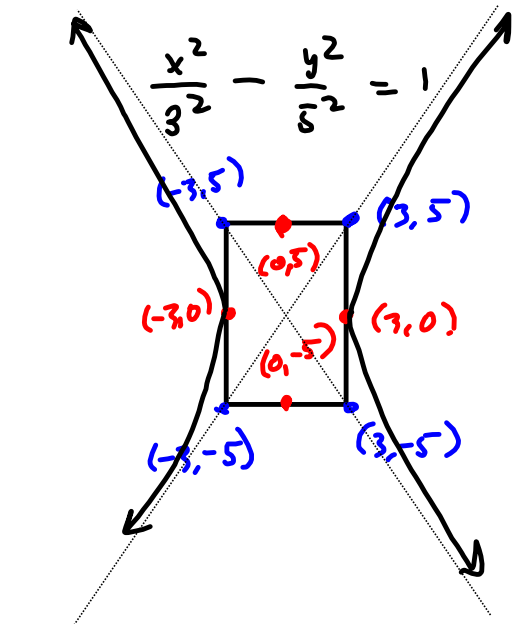
Expnts on Ellipses

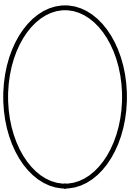
$$\frac{x^2}{3^2} + \frac{y^2}{5^2} = 1$$





$$\frac{x^2}{3^2} - \frac{y^2}{5^2} = 1$$

$$\frac{y^2}{5^2} - \frac{x^2}{5^2} = 1$$



$$5x^2 - 7y^2 = 11$$

$$\frac{5x^2}{11} - \frac{7y^2}{11} = 1$$

$$\frac{x^2}{\left(\frac{11}{5}\right)} - \frac{y^2}{\left(\frac{11}{7}\right)} = 1$$

$$\sqrt{\frac{11}{5}} = \frac{\sqrt{11}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{55}}{5}$$

$$\left(\frac{\sqrt{55}}{5}, \frac{\sqrt{77}}{7}\right)$$

Tangent line: $y = m(x-x_1)+y_1$

$$y = f'(x_1)(x-x_1) + f(x_1) \approx f(x)$$

for $|x-x_1|$ = small.

$f(x_2) - f(x_1) = \Delta y$

$y_2 - y_1 \approx m(x_2-x_1)$

$= f'(x_1)(x_2-x_1)$

$\frac{y_2-y_1}{x_2-x_1} = m$

$y_2-y_1 = m(x_2-x_1)$

$y-y_1 = m(x-x_1)$

$y = m(x-x_1)+y_1$

$\Delta y = f(x_2)-f(x_1) \approx f'(x_1)(x_2-x_1)$

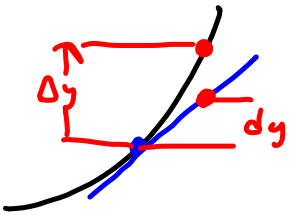
$\Delta y \approx f'(x_1)(x_2-x_1)$

$\Delta y \approx f'(x_1)\Delta x$

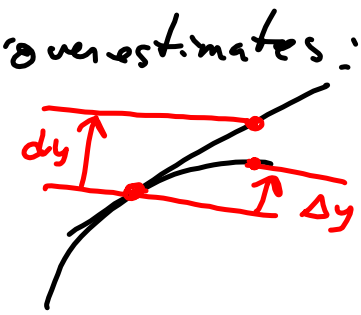
Definition:

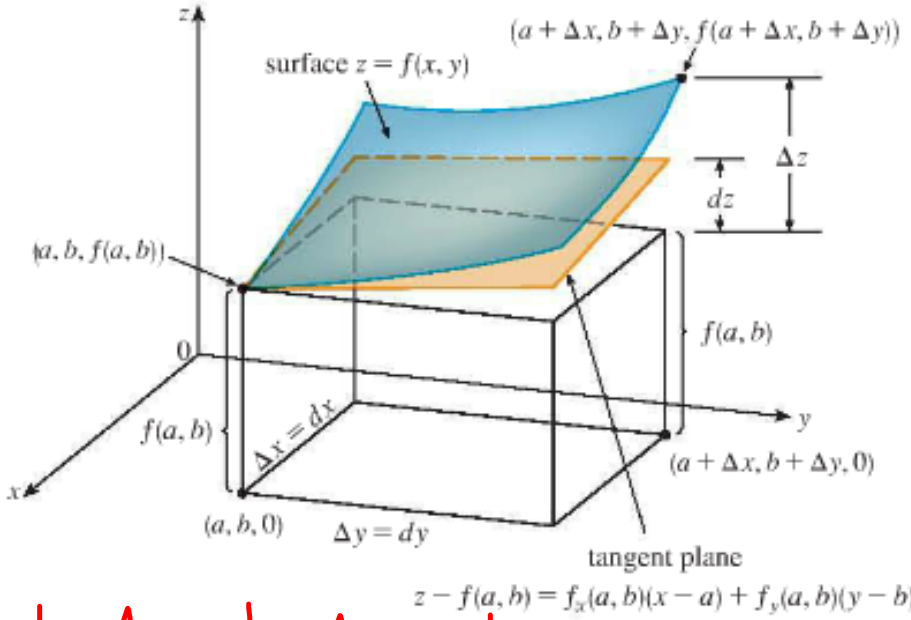
$dy = f'(x_1)\Delta x$ or $f'(x)dx$

dy underestimates Δy:



overestimates:





$dx = \Delta x$, $dy = \Delta y$ $\quad \frac{dx}{\Delta x}$ $\quad \frac{dy}{\Delta y}$

$$z \approx f_x(a,b)(x-a) + f_y(a,b)(y-b) + f(a,b)$$

Tan. Plane

$f_x(a,b)$: Direction vector

$$y \approx f'(a)(x-a) + f(a) \quad \text{Tan. line}$$

$$\Delta z \approx dz = f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

Estimates Change in z.

Soup can. 10 cm high, 4 cm diameter
Top & bottom .1 cm thick.
sides .05 cm
Estimate amt. of metal used to make the can, using differentials.

$V(r,h)$ = Volume of can. $= \pi r^2 h$

$dV = V_r \Delta r + V_h \Delta h$

$= V_r\, dr + V_h\, dh$

$= 2\pi rh\, dr + \pi r^2\, dh$

$= 2\pi(2)(10)(.05) + \pi(2)^2(.2)$

$= 4\pi[.5 + .2] = 2.8\pi$

Actual vol of metal used.

Assume measurements are outer dimensions:

$\pi(2)^2(10) - \pi(1.95)^2(9.8)$

.2, b/c top & bottom,

Vol. of can is inside $\pi(40 - (9.8)(1.95)^2)$ $2.73550\,\pi$

outside vol. of can $\pi((10.2)(2.05)^2 - 40)$