$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2}$$

x before y, above

$\int$ 14.1 #s

$\int$ 14.2 #s

Some $\int$ 14.3 stuff

$\int$ 14.4 #s 2, 5, 7, 12, 19, 25, 28, 32, 42

15–40 Find the first partial derivatives of the function.

30. $F(\alpha, \beta) = \int_\alpha^\beta \sqrt{t^3+1}\, dt$ $= -\int_\beta^\alpha \sqrt{t^3+1}\, dt$

$F_\alpha = \frac{d}{d\alpha}\left[-\int_\beta^\alpha \sqrt{t^3+1}\, dt \right] = \boxed{-\sqrt{\alpha^3+1}}$

$F_\beta = \frac{\partial}{\partial\beta}\left[\int_\alpha^\beta \sqrt{t^3+1}\, dt \right] = \sqrt{\beta^3+1}$

FTC I $\quad \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

32. $f(x, y, z) = x\sin(y - z)$

$f_x = \sin(y-z)$ $\quad y, z$ constant

$f_y = x\cos(y-z)$ $\quad x, z$ ''

$f_z = x(\cos(y-z))(-1)$ $\quad x, y$ ''

47–50 Use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$.

47. $x^2 + 2y^2 + 3z^2 = 1$ **48.** $x^2 - y^2 + z^2 - 2z = 4$

$\frac{\partial z}{\partial x}$: Assume $z = f(x,y)$ so, $\frac{\partial}{\partial x}\left[3z^2\right] = 6z\frac{\partial z}{\partial x}$

$2x + 6zz' = 0$ but $\frac{\partial}{\partial x}\left[2y^2\right] = 0$!

$6zz' = -2x$

$z' = \frac{-2x}{6z} = -\frac{1}{3}\left(\frac{x}{z}\right) = \frac{\partial z}{\partial x} = -\frac{x}{3}z^{-1} = f_x$

$\frac{\partial z}{\partial y}$: $0 + 4y + 6zz' = 0$

$\frac{\partial z}{\partial y}$

$6zz' = -4y$

$z' = -\frac{4y}{6z} = \frac{\partial f}{\partial y} = f_y$

Check: $f_{xy} = \frac{\partial}{\partial y}\left[-\frac{x}{3}z^{-1}\right] = \frac{x}{3}z^{-2}\frac{\partial z}{\partial y} = \left(\frac{x}{3z^2}\right)\left(-\frac{4y}{6z}\right)$

$f_{yx} = \frac{\partial}{\partial x}\left[\frac{-2y}{3z}\right] = \frac{\partial}{\partial x}\left[-\frac{2y}{3}z^{-1}\right]$ $= \left(-\frac{2}{9}\right)\left(\frac{xy}{z^3}\right)$

$= \frac{-2y}{3}(-1z^{-2})\left(\frac{\partial z}{\partial x}\right)$ Ahhh!

$= \left(\frac{2y}{3z^2}\right)\left(-\frac{x}{3}z^{-1}\right) = \left(-\frac{2}{9}\right)\left(\frac{xy}{z^3}\right)$

53–58 Find all the second partial derivatives.

58. $v = e^{xe^y}$

$v = e^{xe^y}$

$= e^{x\cdot 7} = e^{7x} \xrightarrow{\frac{d}{dx}} 7e^{7x}$

$v_x = e^y\left(e^{xe^y}\right) = e^{y+xe^y}$

$v_{xx} = e^y \cdot e^y e^{xe^y} = e^{2y}e^{xe^y} = e^{2y+xe^y}$

$v_{xy} = e^y e^{xe^y} + e^y\left(xe^y\right)e^{xe^y} = e^{xe^y+y} + xe^{2y+xe^y}$

$v_y = xe^y e^{xe^y} = xe^{y+xe^y}$

$v_{yy} = xe^y e^{xe^y} + xe^y\left(xe^y\right)e^{xe^y}$

$v_{yx} = e^y e^{xe^y} + xe^y\left(e^y e^{xe^y}\right)$

$= e^{y+xe^y} + xe^{2y+xe^y}$

If you're doing it right,

$f_{xy} = f_{yx}$

59–62 Verify that the conclusion of Clairaut's Theorem holds, that is, $u_{xy} = u_{yx}$.

59. $u = x^4y^3 - y^4$

60. $u = e^{xy} \sin y$

Take mixed 2nd partials & show they're equal, like my "check" in #58:

$f_{xy} = f_{yx}$ is very good sign you're doing it correctly.