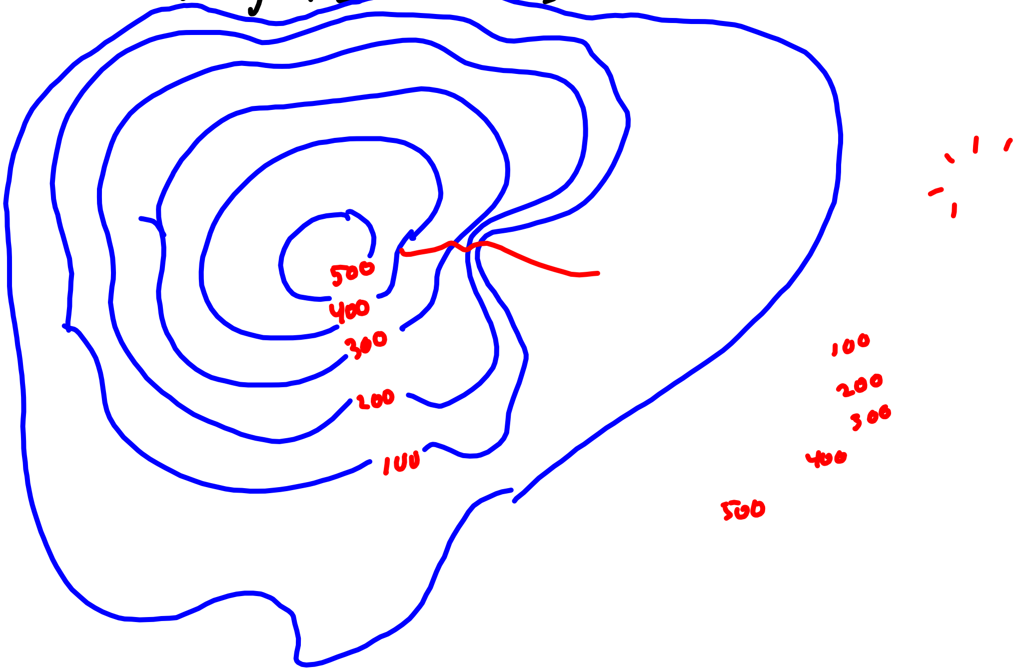
Hypersphere is the surface of the unit "ball."

$$x^2+y^2+z^2+w^2+l^2 \leq 1$$



§ 14.1

Level Curves, Contours

We take horizontal slices.

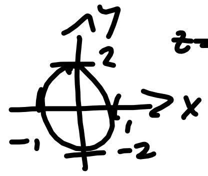
& find their trace.

$$f(x,y) = z = x^2 + \frac{y^2}{4}$$

$z=0$ $\quad x^2 + \frac{y^2}{4} = 0$ $\quad$ is just $(0,0,0)$

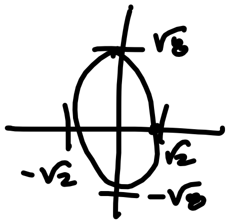
$z=1$ $\quad x^2 + \frac{y^2}{4} = 1$

z=1



$z=2$ $\quad x^2 + \frac{y^2}{4} = 2$

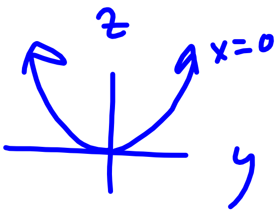
$$\frac{x^2}{2} + \frac{y^2}{8} = 1$$

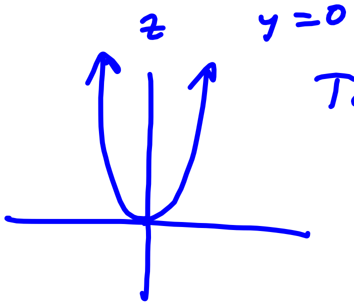


Maybe one or 2 $x=K$, $y=K$ deals.

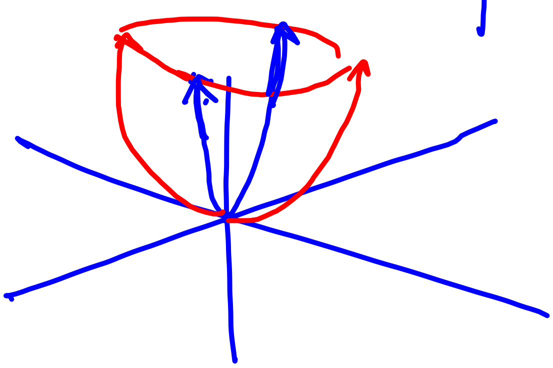
$x=0$

$$z = 0^2 + \frac{y^2}{4} \Rightarrow z = \frac{y^2}{4}$$





Taller parabola



14.1 #s 5, 7, 10, 11, 16, 17, 20, 21, 23, 27, 29-31, 43, 49, 50, 65, 67, 68

S 14.2 #s 5, 8, 13, 16, 25, 39*

Bleah!

$\cancel{\sqrt{25-x^2-y^2} \le 5-x-y}$

#18 Find D & sketch D

$f(x,y) = \sqrt{y} + \sqrt{25-x^2-y^2}$

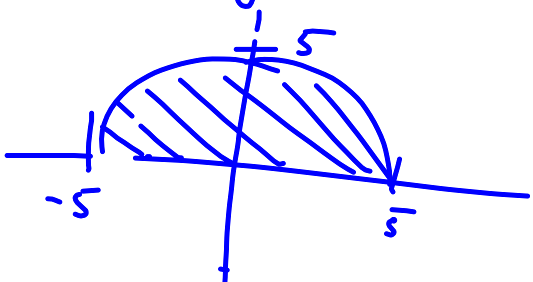
Need:

$y \ge 0$

$25-x^2-y^2 \ge 0$

$25 \ge x^2+y^2$

$x^2+y^2 \le 25$ = the Disk!



Unit ball in $\mathbb{R}^2$: $\sqrt{x^2+y^2} \le 1$

Unit ball in $\mathbb{R}^3$: $\sqrt{x^2+y^2+z^2} \le 1$

$\vdots$

unit Ball in $\mathbb{R}^n$: $\sqrt{x_1^2+x_2^2+\cdots+x_n^2} \le 1$

§14.2

$f(x,y) = z$ A Surface in 3-space.

$\frac{\partial f}{\partial x}$ Hold y constant. Take $\frac{df}{dx}$

$\partial =$ "del"

$f(x,y) = x^2 \sin(xy)$

$\frac{\partial f}{\partial x} = f_x = 2x\sin(xy) + x^2\cos(xy)\cdot y$

Chain rule on the "yx" or the "3x"

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left[\frac{\partial f}{\partial x}\right] = 2\sin(xy) + 2x\cos(xy)\cdot y$$
$$+ 2x\cos(xy)\cdot y - x^2\sin(xy)\cdot y\cdot y$$

$$\frac{\partial f}{\partial x} = f_x = 2x\sin(xy) + x^2\cos(xy)\cdot y$$

$$f_x = 2x\sin(xy) + x^2\cos(xy)\cdot y$$

$$f_{xy} = 2x\cos(xy)\cdot x - x^2\sin(xy)y + x^2\cos(xy)\cdot 1$$

$$= \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y\,\partial x}$$

continuous derivatives

Clairaut's Theorem: If things go smooth

$f_{xy} = f_{yx}$ Mixed 2nd partials are the same, regardless of the order in which they were calculated

$$f(x,y) = x^2 \sin(xy) \Longrightarrow$$

$$f_y = x^2 \cos(xy) \cdot x = x^3 \cos(xy)$$

Sucks

$$f_{yx} = 3x^2 \cos(xy) - x^3 \sin(xy) \cdot y$$

$$f_{xy} = 2x \cos(xy) \cdot x - x^2 \sin(xy) y + x^2 \cos(xy) \cdot 1$$

$$f(xy) = x^2 y^3 + y^2 x$$

$$f_x = 2xy^3 + y^2 \qquad f_y = 3x^2y^2 + 2xy$$

$$f_{xy} = 6xy^2 + 2y \qquad f_{yx} = 6xy^2 + 2y$$

$f = x^2 \sin(3x) \Rightarrow$

$\frac{df}{dx} = 2x \sin(3x) + x^2 (\cos(3x)) \cdot 3$

Lots of Theory that says what we're doing, in practice.

What's going on?

$\frac{\partial f}{\partial x}$ = tangent of the trace in the $y = K$ plane

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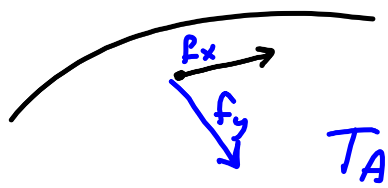
Fig 1



f_x & f_y at a given point defines the TANGENT Plane.