Test 1 Monday.

Recall §10.2

$$s = L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_{\alpha}^{\beta} ds$$

Bump it up to 3-D.

$\bar{a} \cdot \bar{a} = |\bar{a}|^2$

$\boxed{2}$ $$L = \int_{\alpha}^{\beta} \sqrt{f'(t)^2 + g'(t)^2 + h'(t)^2}\, dt$$

where $\bar{r}(t) = \langle f(t), g(t), h(t) \rangle$

$$= \int_{\alpha}^{\beta} |\bar{r}'(t)|\, dt$$

Increasing Func. of t.

$\boxed{6}$ $$L(t) = s(t) = \int_{a}^{t} |\bar{r}'(u)|\, du \Rightarrow$$

$$\frac{ds}{dt} = s'(t) = |\bar{r}'(t)| \qquad \text{by FTC I.}$$

$$\frac{d}{dt} \int_{0}^{t} x^2\, dx = t^2$$

$$\frac{d}{dt} \int_{0}^{t^3} x^2\, dx = (t^3)^2 (3t^2)$$

Recall $\bar{T} = \dfrac{\bar{r}'}{|\bar{r}'|}$

Curvature:

$$\kappa = \left| \frac{d\bar{T}}{ds} \right| = \left| \frac{d\bar{T}/dt}{ds/dt} \right| \; *$$

derivative of $\bar{T}$ w.r.t. s.

$$* \quad \frac{d\bar{T}}{dt} = \frac{d\bar{T}}{ds} \cdot \frac{ds}{dt}$$

(Assume $t = t(s)$
The inverse of $s = s(t)$
"$s = s$ of t")

$$\Rightarrow \frac{\frac{d\bar{T}}{dt}}{\frac{ds}{dt}} = \frac{d\bar{T}}{ds}$$

"Kappa"

$\boxed{T10}$ $\kappa(t) = \dfrac{|\bar{T}'(t)|}{|\bar{r}'(t)|} = \dfrac{|\bar{r}' \times \bar{r}''|}{|\bar{r}'|^3}$

EZ for calculations. Impossible to remember.

$\bar{T} = \dfrac{\bar{r}'}{|\bar{r}'|} \Rightarrow \bar{T}' =$ ouch!

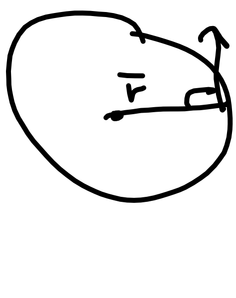
$\bar{T}' = \left(|\bar{r}'|\,\bar{r}'\right)'$

Differentiating <u>THIS</u> is hard!

<u>T10 Proof</u>: Stuff they left out,

⋮ "Since $\bar{T}$ is $\perp$ to $\bar{T}'$"

why? Because $|\bar{T}| =$ constant,



$$|\bar{r}' \times \bar{r}''|$$

$$\bar{r}'' = \left(\frac{ds}{dt}\bar{T}\right)' = \frac{d^2s}{dt^2}\bar{T} + \frac{ds}{dt}\bar{T}'$$

$$= \left|\frac{ds}{dt}\bar{T} \times \left(\frac{d^2s}{dt^2}\bar{T} + \frac{ds}{dt}\bar{T}'\right)\right|$$

$$\left|\frac{ds}{dt}\bar{T} \times \frac{d^2s}{dt^2}\bar{T} + \frac{ds}{dt}\bar{T} \times \frac{ds}{dt}\bar{T}'\right|$$

$$= \left|\cancel{\left(\frac{ds}{dt}\right)\left(\frac{d^2s}{dt^2}\right)\bar{T} \times \bar{T}} + \left(\frac{ds}{dt}\right)^2 \bar{T} \times \bar{T}'\right|$$

$$\bar{a} \times \bar{b} = |\bar{a}||\bar{b}|\sin\theta \quad T\ 12.4.9$$

$$\bar{T} \times \bar{T} = |\bar{T}||\bar{T}|\sin\theta = 0$$

$$= \left|\left(\frac{ds}{dt}\right)^2 \bar{T} \times \bar{T}'\right|$$

$$\Rightarrow |\bar{r}' \times \bar{r}''| = \left(\frac{ds}{dt}\right)^2 |\bar{T} \times \bar{T}'|$$

$$= \left(\frac{ds}{dt}\right)^2 \left||\bar{T}||\bar{T}'|\sin\theta\right| \qquad |\bar{r}'|$$

$$= \left(\frac{ds}{dt}\right)^2 |\bar{T}|\,|\bar{T}'| = |\bar{r}' \times \bar{r}''|$$

$$\Rightarrow |\bar{T}'| = \boxed{\frac{|\bar{r}' \times \bar{r}''|}{|\bar{r}'|^2}} \qquad \text{d so}$$

$$\kappa = \frac{|\bar{T}'|}{|\bar{r}'|} = \frac{|\bar{r}' \times \bar{r}''|}{|\bar{r}'|^3}$$

In 2-D, when $y = f(x)$,

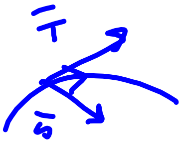
$\vec{r} = \langle x, f(x) \rangle$

& T10 becomes

$$\boxed{\kappa(x) = \frac{|f''(x)|}{(1 + f'(x)^2)^{3/2}}}$$

NORMAL

$$\bar{N} = \frac{\bar{T}'(t)}{|\bar{T}'(t)|}$$



$\bar{T} \perp \bar{T}'$ b/c

$|\bar{T}|$ = constant.

$\bar{N} \perp \bar{T}$, since

$\bar{N} \parallel \bar{T}'$

$\bar{B} = \bar{T} \times \bar{N}$ is orthog to both

= Binormal Vector.

§ 13.3 #s 1, 4, 7, 10, 13, $\overset{?}{15}$, 17, 21, 28, 47, 50