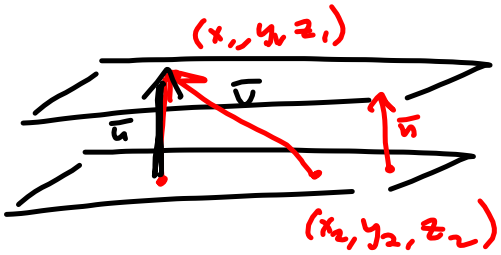
§12.5 #75 §13.1 there's a general talk

Show that the distance between

$ax+by+cz+d_1=0$

$ax+by+cz+d_2=0$

is $\dfrac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$



$\bar{u}=\text{proj}_{\bar{n}}\bar{v}$

$\bar{v}=\langle x_1-x_2, y_1-y_2, z_1-z_2\rangle$

want

$|\text{proj}_{\bar{n}}\bar{v}|$

$=|\text{comp}_{\bar{n}}\bar{v}|$

$=\left|\dfrac{\bar{n}\cdot\bar{v}}{|\bar{n}|}\right|$

$|\bar{n}\cdot\bar{v}|=|a(x_1-x_2)+b(y_1-y_2)+c(z_1-z_2)|$

$=|ax_1-ax_2+by_1-by_2+cz_1-cz_2|$

$=|ax_1+by_1+cz_1-(ax_2+by_2+cz_2)|$

$=|-d_1-(-d_2)|$

$=|-d_1+d_2|=|d_2-d_1|=|d_1-d_2|$, so

$\text{distance}=|\text{comp}_{\bar{n}}\bar{v}|=\dfrac{|d_1-d_2|}{|\bar{n}|}$

$=\dfrac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$ ∎