Homework:

Due 2 lecs after we finish covering the material.

§12.4 #46b,

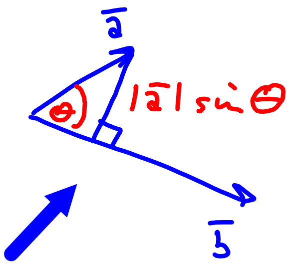
Last time, asked for area of a triangle, but stopped @ area of the corresponding parallelogram, $5\sqrt{82}$, but forgot to divide by 1/2

Cross Product: Slick way:

$\bar{a} = \langle 1,2,3 \rangle$, $\bar{b} = \langle -1,1,3 \rangle$

$$\begin{array}{rc} \bar{a} & \langle 1, 2, 3 \rangle \\ \times\, \bar{b} & \langle -1, 1, 3 \rangle \\ \hline \bar{a} \times \bar{b} & \langle 3, -6, 3 \rangle \end{array}$$

+, −, + thing going on, here.



orthogonal

$\boxed{T8}$ $\bar a \times \bar b \perp$ to $\bar a, \bar b$.

$\boxed{T9}$ $|\bar a \times \bar b| = |\bar a||\bar b|\sin\theta$

$\boxed{Pf}$ $|\bar a \times \bar b|^2 =$ ... and then a miracle occurs...

$$(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2) - (a_1b_1 + a_2b_2 + a_3b_3)^2$$

$$= |\bar a|^2|\bar b|^2 - (\bar a \cdot \bar b)^2$$

$$= |\bar a|^2|\bar b|^2 - |\bar a|^2|\bar b|^2\cos^2\theta$$

$$= |\bar a|^2|\bar b|^2(1 - \cos^2\theta)$$

$$= |\bar a|^2|\bar b|^2\sin^2\theta \implies$$

(Sum: $|\bar a \times \bar b|^2 = |\bar a|^2|\bar b|^2\sin^2\theta$)

$$|\bar a \times \bar b| = \sqrt{|\bar a|^2|\bar b|^2\sin^2\theta}$$

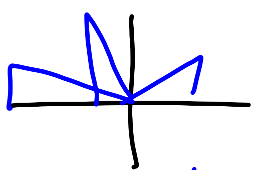
$$= |\bar a||\bar b||\sin\theta|$$

$$= |\bar a||\bar b|\sin\theta$$

∎

$\sqrt{x^2} = |x|$

$(\sqrt{x})^2 = x$



$\sin\theta > 0$ when $\theta \in (0, \pi)$ which it is.

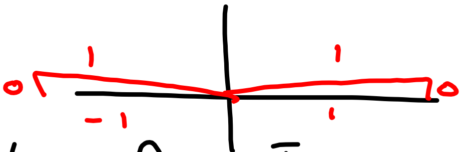
So, $\sin\theta = \dfrac{|\bar a \times \bar b|}{|\bar a||\bar b|}$

$$\cos\theta = \frac{\bar a \cdot \bar b}{|\bar a||\bar b|} \implies \bar a \cdot \bar b = |\bar a||\bar b|\cos\theta$$

Recall $\bar{a} \perp \bar{b}$ iff $\bar{a} \cdot \bar{b} = 0$

$\boxed{T10}$ New! $\bar{a} \parallel \bar{b}$ iff $\bar{a} \times \bar{b} = 0$

$\sin\theta = 0$



$\boxed{T11}$ Properties of $\bar{a} \times \bar{b}$

① $\bar{a} \times \bar{b} = -\bar{b} \times \bar{a}$

② $(c\bar{a}) \times \bar{b} = c(\bar{a} \times \bar{b})$ "Respects scalar multiples."

③ $\bar{a} \times (\bar{b} + \bar{c}) = \bar{a} \times \bar{b} + \bar{a} \times \bar{c}$ "Distributive"

⑤ * $\bar{a} \cdot (\bar{b} \times \bar{c}) = (\bar{a} \times \bar{b}) \cdot \bar{c}$

⑥ $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c}$

* Scalar Triple product. Vol. of parallelepiped.

$$\langle 5, -30, 10 \rangle \times \langle 1, 2, 3 \rangle$$

Same as

5 times

$$\langle 1, -6, 2 \rangle \times \langle 1, 2, 3 \rangle$$

$$5 \langle -22, 1, 8 \rangle$$

$$\langle 5, -30, 10 \rangle \times \langle 1, 2, 3 \rangle$$

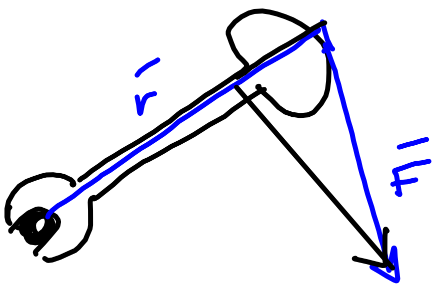
Same as

5 times

$$\langle 1, -6, 2 \rangle \times \langle 1, 2, 3 \rangle$$

$$5 \langle -22, 1, 8 \rangle$$

Torque



Torque, tendency to rotate, is

$$\bar{\tau} = |\bar{r} \times \bar{F}|$$

Only the component of $\bar{F}$ that's $\perp$ to $\bar{r}$ makes any contribution. (to the tendency to rotate.)

($|\bar{u} \times \bar{v}| = 0$ iff $\bar{u} \parallel \bar{v}$.)