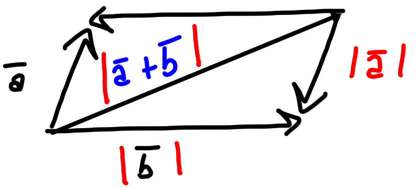
§12.2 questions ?

§12.3? 62

§12.4 ?

#62 (a) Picture for $|\bar{a}+\bar{b}| \le |\bar{a}|+|\bar{b}|$



Obviously,
$|a+b| \le |\bar{a}| + |\bar{b}|$

(b) Cauchy-Schwartz Says

$$|\bar{a}\cdot\bar{b}| \le |\bar{a}||\bar{b}|$$

Always keeps popping up, forever.

$$\left|\int f(x)g(x)\,dx\right| \le \sqrt{\int |f(x)|^2 dx}\sqrt{\int |g(x)|^2 dx}$$

Now, memory fades.

2-D version

want $|a_1b_1 + a_2b_2| \le \sqrt{a_1^2+a_2^2}\sqrt{b_1^2+b_2^2}$

Triangle Inequality

$$|\bar{a}+\bar{b}| \le |\bar{a}| + |\bar{b}|$$

$$\sqrt{(a_1+b_1)^2 + (a_2+b_2)^2} \le \sqrt{a_1^2+a_2^2} + \sqrt{b_1^2+b_2^2}$$

Always

$\langle a_1, a_2\rangle + \langle b_1, b_2\rangle$

$= \langle a_1+b_1, a_2+b_2\rangle$ ξ

$|\bar{a}+\bar{b}| =$

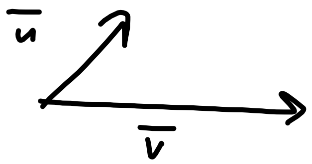
Punt !

Sorry. Not able to express this, live.

Vid on #62

Big Results on Cross Product

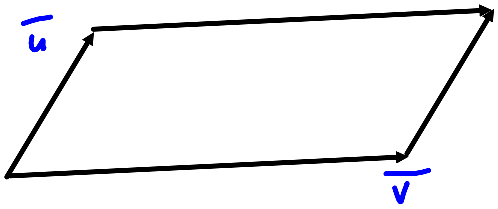
$\bar{u} \times \bar{v}$ is $\perp$ to both $\bar{u}$ and $\bar{v}$!



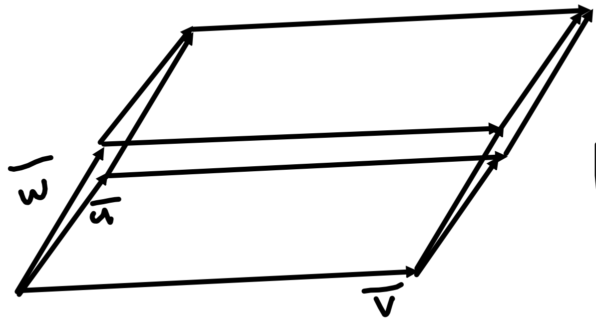
Right-hand rule,
Point wrist in direction of $\bar{u}$ and curl fingers in direction of $\bar{v}$.

$\bar{u} \times \bar{v}$ is in the direction your thumb is pointing.

Other big result: Area & volume.



$|\bar{u} \times \bar{v}| =$ Area of the parallelogram



The parallelepiped has volume

$|\bar{u} \cdot (\bar{v} \times \bar{w})| = |\bar{v} \cdot (\bar{u} \times \bar{w})|$

$= |\bar{w} \cdot (\bar{v} \times \bar{u})|$, etc.

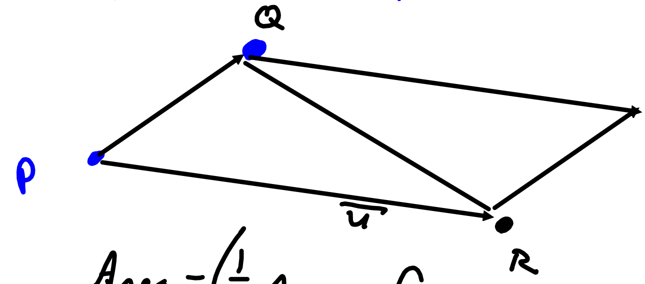
is "scalar triple product.

Proofs in book.

Proofs are good for video.

Find area of the triangle

$P(1,4,6)$, $Q(-2,5,-1)$, $R(1,-1,1)$



Area $= \left(\frac{1}{2}\text{ Area of parallelogram}\right)$

$\overrightarrow{PR} = \langle 1-1, -1-4, 1-6\rangle$ points at R

$= \langle 0, -5, -5\rangle = \bar{u}$

$\overrightarrow{PQ} = \langle -2-1, 5-4, -1-6\rangle =$

$= \langle -3, 1, -7\rangle = \bar{v}$

Area $= |\bar{u} \times \bar{v}|$

$=$ Abs val of

$$\bar{u} \times \bar{v} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & -5 & -5 \\ -3 & 1 & -7 \end{vmatrix}$$

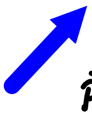
$$= +\left((-5)(-7) - (1)(-5)\right)\bar{i}$$
$$- \left((0)(-7) - (-3)(-5)\right)\bar{j}$$
$$+ \left((0)(1) - (-3)(-5)\right)\bar{k}$$

$$= (35+5)\bar{i} - (-15)\bar{j} + (-15)\bar{k}$$

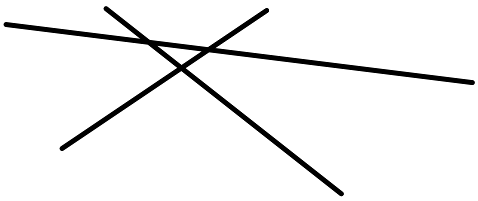
$40\bar{i} + 15\bar{j} - 15\bar{k} = \langle 40, 15, -15\rangle = 5\langle 8, 3, -3\rangle$

So $|\bar{u} \times \bar{v}| = 5\sqrt{8^2 + 3^2 + (-3)^2}$

$= 5\sqrt{82}$



$\overrightarrow{PQ}$



$P(1,2)$ $R(-1,1)$

$P\ \langle 1,2\rangle\ ,\ \langle -1,1\rangle$