12.3 Homework

Posted Jan 22, 2016 9:05 AM

S 12.3 #s 1, 3, 7, 9, 13, 15, 19, 23, ~~24~~, 25, 29, 39, 44, 45, 49, 58*, (61 optional), 62

* #58 I want you to sweat over.

#61 result is used for #62. If you want to prove #61, to yourself, that's fine.

S 12.4 #s 1, 6, 9, 10, 13, 15, 17, 19, 27, 29, 35, 37, 39, 41, 45*, 46*

* Distance from point to line and pt to plane. Pretty important stuff.

§12.3 cnt'd

Distance between 2 "spheres" not circles.

$$\bar{u}\cdot\bar{v} = u_1v_1 + u_2v_2$$

Properties of Dot Product

① $\bar{u}\cdot\bar{u} = |\bar{u}|^2$

pf

$$\bar{u}\cdot\bar{v} = u_1u_1 + u_2u_2 = u_1^2 + u_2^2$$

$$|\bar{u}|^2 = \left(\sqrt{u_1^2+u_2^2}\right)^2$$

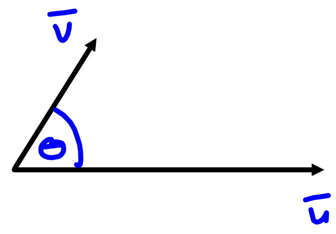
② $\bar{u}\cdot\bar{v} = \bar{v}\cdot\bar{u}$

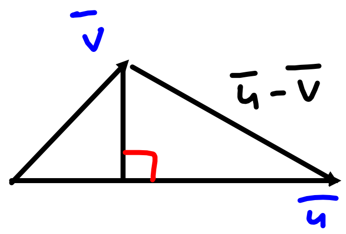
③ $\bar{u}\cdot(\bar{v}+\bar{w}) = \bar{u}\cdot\bar{v} + \bar{u}\cdot\bar{w}$

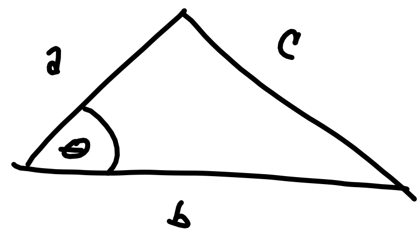
④ $(c\,\bar{u})\cdot\bar{v} = c(\bar{u}\cdot\bar{v})$

⑤ $\bar{0}\cdot\bar{u} = 0$

Cosine of angle between 2 vectors.







$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$|\bar u - \bar v|^2 = |\bar v|^2 + |\bar u|^2 - 2|\bar v||\bar u|\cos\theta$$

$$(\bar u - \bar v)\cdot(\bar u - \bar v) =$$

$$\bar u\cdot\bar u - \bar u\cdot\bar v - \bar v\cdot\bar u + \bar v\cdot\bar v =$$

$$|\bar u|^2 - 2\bar u\cdot\bar v + \cancel{|\bar v|^2} = \cancel{|\bar v|^2} + |\bar u|^2 - 2|\bar v||\bar u|\cos\theta$$

$$-2\bar u\cdot\bar v = -2|\bar v||\bar u|\cos\theta$$

T3 says $\longrightarrow$ $\bar u\cdot\bar v = |\bar v||\bar u|\cos\theta$

T6 says $\boxed{\cos\theta = \dfrac{\bar u\cdot\bar v}{|\bar u||\bar v|}}$

by which we find θ via $\cos^{-1}(\cos\theta)$

$$|\bar u - \bar v|^2 = (\bar u - \bar v)\cdot(\bar u - \bar v)$$

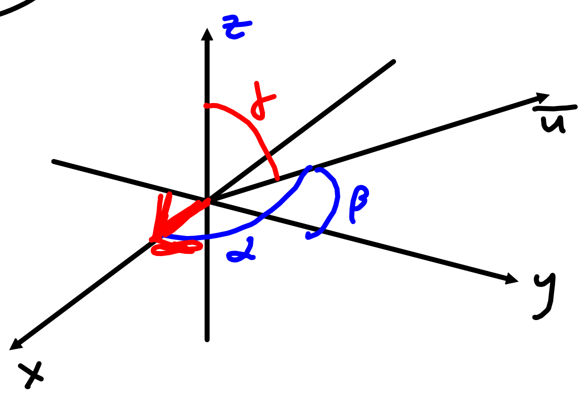
Direction Angles

α = angle from $\bar{u}$ to x-axis

β = " " $\bar{u}$ " y-axis

γ = " " " " z-axis

Pg 827



$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|}$$

$$\alpha: \quad \cos\alpha = \frac{\bar{u}\cdot\bar{i}}{|\bar{u}||\bar{i}|} = \frac{u_1}{|\bar{u}|}$$

Direction cosines

$$\bar{u}\cdot\bar{i} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = u_1$$

$$\beta: \quad \cos\beta = \frac{u_2}{|\bar{u}|}$$

$$\gamma: \quad \cos\gamma = \frac{u_3}{|\bar{u}|}$$

These angles ALL live in $[0, \pi]$

$\alpha, \beta, \gamma \in [0, \pi]$

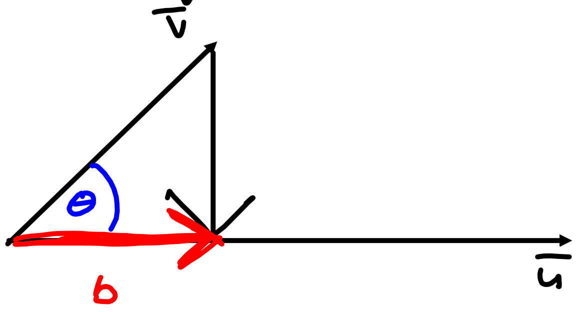
FACTS

$$\bar{u} = \langle |\bar{u}|\cos\alpha, |\bar{u}|\cos\beta, |\bar{u}|\cos\gamma \rangle$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Projections

$\boxed{\text{Scalar Component of } \bar{v} \text{ on } \bar{u}}$ is the length of its shadow on $\bar{u}$



$$\frac{b}{|\bar{v}|} = \cos\theta$$

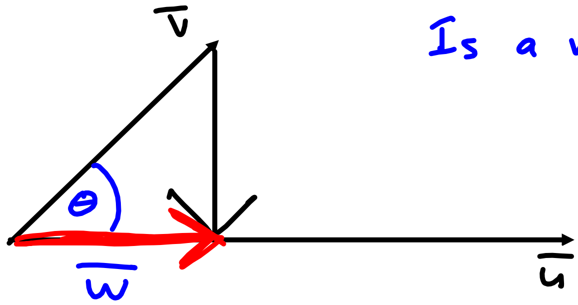
Is a magnitude

$$b = |\bar{v}|\cos\theta = \left(|\bar{v}|\right)\left(\frac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|}\right)$$

$$= \frac{\bar{u}\cdot\bar{v}}{|\bar{u}|} = \text{comp}_{\bar{u}}\bar{v}$$

Vector Component of $\bar{v}$ on $\bar{u}$

$$\bar{w} = \text{proj}_{\bar{u}}\bar{v} = \left(\frac{\bar{u}\cdot\bar{v}}{|\bar{u}|}\right)\left(\frac{\bar{u}}{|\bar{u}|}\right) = \left(\frac{\bar{u}\cdot\bar{v}}{|\bar{u}|^2}\right)\bar{u}$$



Is a vector

Unit Vector in direction of $\bar{u}$

$$\frac{1}{|\bar{u}|}\bar{u}$$ is its length

$$\left|\frac{1}{|\bar{u}|}\bar{u}\right| = \frac{1}{|\bar{u}|}|\bar{u}| = 1$$

Determinant

$$\begin{vmatrix} \bar{i} & \bar{j} & k \\ 2 & 3 & 4 \\ \bar{5} & 6 & 7 \end{vmatrix}$$

$$= \Big((3)(7) - (4)(6)\Big)\, i$$

$$- \Big((2)(7) - (5)(4)\Big)\, j \qquad -(14 - 20)\, j = +6j$$

$$+ \Big((2)(6) - (5)(3)\Big)\, k \qquad = -3\bar{i} + 6\bar{j} - 3\bar{k}$$

$$= \langle -3, -3, -3 \rangle$$

Is how we do CROSS PRODUCT using $\bar{i}, \bar{j}, \bar{k}$