Distance between 2 circles

$x^2 + y^2 + z^2 = 4$, $r_2 = 2$

$x^2+y^2+z^2 = 4x+4y+4z-11$

$(h_2, k_2, l_2) = (0,0,0) = B$

$x^2 - 4x + 2^2 + y^2 - 4y + 2^2 + z^2 - 4z + 2^2 = -11 + 12 = 1$

$(x-2)^2 + (y-2)^2 + (z-2)^2 = 1$

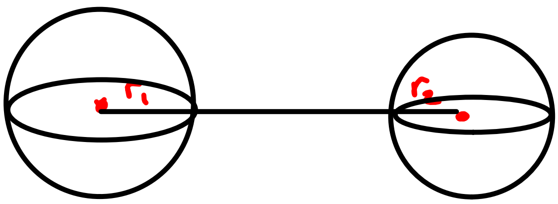
$(h_1, k_1, l_1) = (2,2,2) = A$, $r_1 = 1$

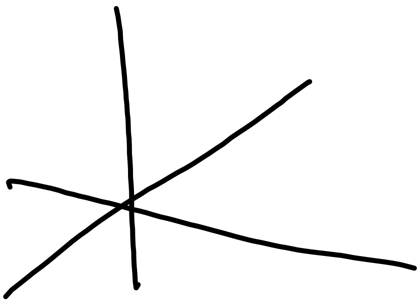
$d(A,B) = \sqrt{2^2+2^2+2^2} = \sqrt{12} = 2\sqrt{3}$

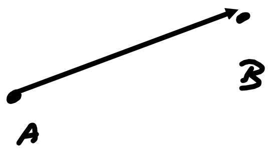
$r_1 + r_2 = 2+1 = 3$

$d(A,B) - 3 = \boxed{2\sqrt{3} - 3}$

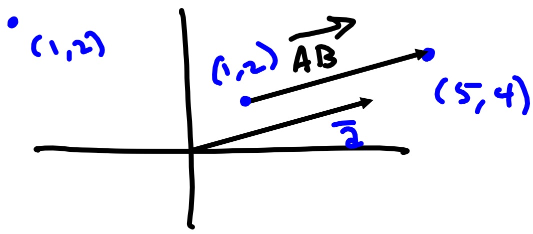
S' 12.2 #s 1, 5, 8, 9, 15, 18, 21, 23, 24, 26, 27, 29, 30, 36





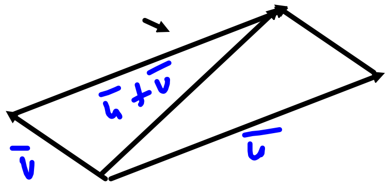


Directed segment from A (a_1, a_2, a_3) & B (b_1, b_2, b_3) is $\overrightarrow{AB}$ "=" $\bar{a}$ is the displacement vector



$\bar{a} = \langle 4-2, 5-1 \rangle$

Adding Vectors



$\bar{u} = \langle u_1, u_2 \rangle$, $\bar{v} = \langle v_1, v_2 \rangle$

$\bar{u} + \bar{v} =$

$\langle u_1 + v_1, u_2 + v_2 \rangle = \bar{w}$

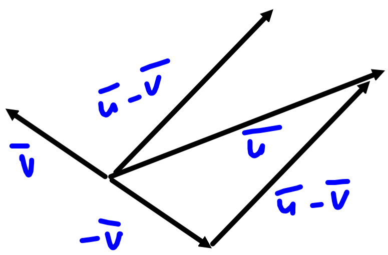
"Parallelogram Rule"

"Triangle Law"

Subtracting Vectors

$\bar{u} - \bar{v} = \bar{u} + (-\bar{v})$

$= \langle u_1 - v_1, u_2 - v_2, u_3 - v_3 \rangle$



$-\bar{v} = -1 \langle v_1, v_2, v_3 \rangle$

$= \langle -v_1, -v_2, -v_3 \rangle$

Scalar Multiples "Scale"

$c \in \mathbb{R}$, $\bar{u} \in \mathbb{R}^n$

then $c\bar{u}$ is a vector in the same direction as $\bar{u}$, but c times its magnitude

$|\bar{u}|$

So $|c\bar{u}| = c|\bar{u}|$

E $\bar{u} = \langle 1,2 \rangle$, $c = 7$

$c\bar{u} = 7\langle 1,2 \rangle = \langle 7,14 \rangle$

Properties :

* ① $\bar{u} + \bar{v} = \bar{v} + \bar{u}$ commutative

* ② $\bar{u} + (\bar{v} + \bar{w}) = (\bar{u} + \bar{v}) + \bar{w}$ associative

* ③ $\bar{u} + \bar{0} = \bar{0} + \bar{u} = \bar{u}$ $\bar{0}$ exists & does what one hopes.

④ $\bar{u} + (-\bar{u}) = \bar{0}$

$\bar{0} \neq 0$

$\langle 0, 0 \rangle \neq 0$

* ⑤ $c(\bar{u} + \bar{v}) = c\bar{u} + c\bar{v}$

* ⑥ $(c+d)\bar{u} = c\bar{u} + d\bar{u}$

Distributive

⑦ $(cd)\bar{u} = c(d\bar{u})$

* ⑧ $1\bar{u} = \bar{u}$

* Vector Space over a Field.

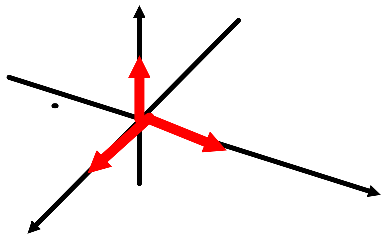
$\mathbb{R}^n$

$\mathbb{R}$

Standard (Canonical) Basis

$\bar{e}_1 = \bar{i} = \langle 1,0,0 \rangle$, $\bar{j} = \bar{e}_2 = \langle 0,1,0 \rangle$, $\bar{k} = \langle 0,0,1 \rangle = \bar{e}_3$

These are all unit vectors!
Their lengths are 1.



Easy to decompose any vector into a linear combo of $\bar{i}, \bar{j}, \bar{k}$.

$\bar{u} = \langle 5, 7, 8 \rangle = 5\bar{i} + 7\bar{j} + 8\bar{k}$

$= 5\langle 1,0,0 \rangle + 7\langle 0,1,0 \rangle$

$+ 8\langle 0,0,1 \rangle$

§12.3 Dot Product.

I'll post an assignment, tomorrow morning

$\bar{u} = \langle u_1, u_2 \rangle$ & $\bar{v} = \langle v_1, v_2 \rangle$, then

$\bar{u} \cdot \bar{v} = u_1 v_1 + u_2 v_2$

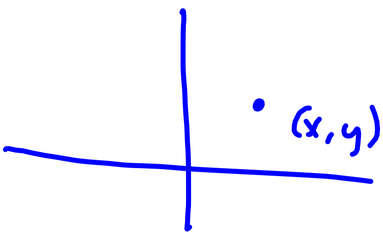
is a function from $\underline{\mathbb{R}^2 \times \mathbb{R}^2 \text{ into } \mathbb{R}^1}$

$((a,b), (c,d))$

$f(x) : \mathbb{R}^1 \longrightarrow \mathbb{R}^1$

$f(x) = x^2$

$f(x,y) = x^2 y$ $\quad f : \mathbb{R}^1 \times \mathbb{R}^1 \longrightarrow \mathbb{R}^1$



$\mathbb{R}^2 = \mathbb{R}^1 \times \mathbb{R}^1$