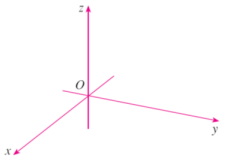
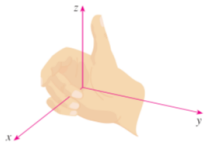
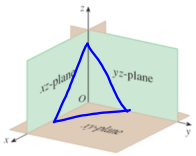


Coordinate axes
Figure 1



Right-hand rule
Figure 2



Coordinate planes
Figure 3(a)

Jan 14-7:31 PM

Calculus III
Generalize $\frac{dy}{dx}$

Semi-Flipped.
Happy to make videos for questions.
4 or 5 tests. Last Test is "Final"
Last test (5) is Ch 16.
Any test can include stuff from previous tests.
(Comprehensive)
Questions class struggled with on previous tests. often these show up as Bonus Probs.

Semi-Flipped. Some Live. Some Video.
In-Class Mostly working and asking questions.
Lowest test counts $\frac{1}{2}$
Remove some pressure.

60% Tests ?
40% Homework ?
T. forgot!

Jan 19-12:13 PM

Look at any bottom corner of a room and call the corner the origin.

The wall on your left is in the *xz*-plane, the wall on your right is in the *yz*-plane, and the floor is in the *xy*-plane.

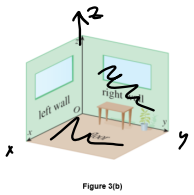


Figure 3(b)

Jan 14-7:39 PM

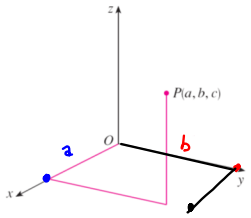
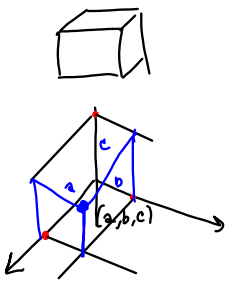


Figure 4



Jan 14-7:44 PM

If we drop a perpendicular from *P* to the *xy*-plane, we get a point *Q* with coordinates (*a*, *b*, 0) called the **projection** of *P* onto the *xy*-plane.

Similarly, *R*(0, *b*, *c*) and *S*(*a*, 0, *c*) are the projections of *P* onto the *yz*-plane and *xz*-plane, respectively.

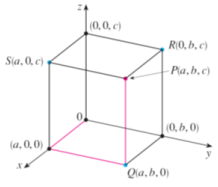


Figure 5

Jan 14-7:52 PM

As numerical illustrations, the points (–4, 3, –5) and (3, –2, –6) are plotted in Figure 6.

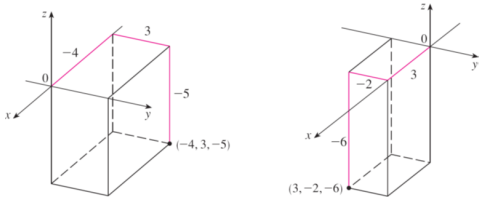


Figure 6

Jan 14-7:57 PM

The Cartesian product $\mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x, y, z) | x, y, z \in \mathbb{R}\}$ is the set of all ordered triples of real numbers and is denoted by $\mathbb{R}^3$.

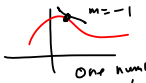
We have given a one-to-one correspondence between points P in space and ordered triples (a, b, c) in $\mathbb{R}^3$. It is called a **three-dimensional rectangular coordinate system**.

Every pt. has (x,y,z) - triple

Every (x,y,z) has a pt.

In two-dimensional analytic geometry, the graph of an equation involving x and y is a curve in $\mathbb{R}^2$.

In three-dimensional analytic geometry, an equation in x, y, and z represents a *surface* in $\mathbb{R}^3$.



Analog of the line is a surface

One number gives rate of change for curves

for a surface? $\frac{dy}{dx} \rightsquigarrow \frac{\partial y}{\partial x}$ partial derivative

$y = f(x) \rightsquigarrow z = f(x,y)$

$\frac{dy}{dx}$ $\frac{dz}{dx}, \frac{dz}{dy}$

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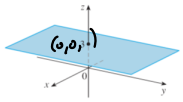
The equation $z = 3$ represents the set $\{(x, y, z) | z = 3\}$, which is the set of all points in $\mathbb{R}^3$ whose z-coordinate is 3.

This is the horizontal plane that is parallel to the xy–plane and three units above it as in Figure 7(a).

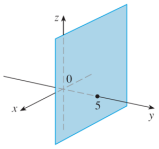


$z = 3$, a plane in $\mathbb{R}^3$

Figure 7(a)

Jan 14-7:58 PM

The equation $y = 5$ represents the set of all points in $\mathbb{R}^3$ whose y-coordinate is 5. This is the vertical plane that is parallel to the xz-plane and five units to the right of it as in Figure 7(b).



$y = 5$, a plane in $\mathbb{R}^3$

Figure 7(b)

Jan 14-8:02 PM

Distance Formula in Three Dimensions The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Generalizes from Pythagorus nicely.

$\sqrt{(x-1)^2 + (y+3)^2 + (z-5)^2} = 25$

Sphere of radius 25 centred @ $(1,-3,5)$

Jan 14-8:03 PM

Find an equation of a sphere with radius r and center $C(h, k, l)$.

Solution:

By definition, a sphere is the set of all points $P(x, y, z)$ whose distance from C is r. (See Figure 12.)

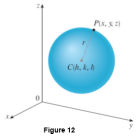


Figure 12

P is on the sphere if and only if $|PC| = r$.

Equation of a Sphere An equation of a sphere with center $C(h, k, l)$ and radius r is

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$$

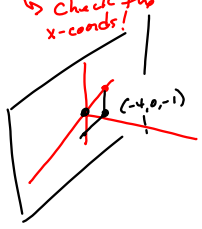
In particular, if the center is the origin O, then an equation of the sphere is

$$x^2 + y^2 + z^2 = r^2$$

Jan 14-8:07 PM

3. Which of the points $A(-4, 0, -1)$, $B(3, 1, -5)$, and $C(2, 4, 6)$ is closest to the yz-plane? Which point lies in the xz-plane?

A: Check the x-coords! $y=0$!



C is closest to yz-plane

A is in the xz-plane.

4. What are the projections of the point (2, 3, 5) on the xy-, yz-, and xz-planes? Draw a rectangular box with the origin and (2, 3, 5) as opposite vertices and with its faces parallel to the coordinate planes. Label all vertices of the box. Find the length of the diagonal of the box.

Jan 14-8:19 PM