203 §14.4 #s 2, 5, 7, 12, 19, 25, 28, 32, 42

#s 1-6 Find eq'n of tan plane to given surface @ given point:

(2) $z = 3(x-1)^2 + 2(y+3)^2 + 7$ @ $(2,-2,12)$

$z_x = 2(x-1) \Longrightarrow z_x(2,-2,12) = 2(1) = 2 = z_x$

$z_y = 2(y+3) \Longrightarrow z_y(2,-2,12) = 2(5) = 10.$

$z = f_x(x-x_0) + f_y(y-y_0) + z_0$

$$\boxed{z = 2(x-2) + 10(y+2) + 12}$$

Other ways of seeing it. More vector-style:

$2(x-2) + 10(y+2) - (z-12) = 0$

$\bar{n} = \langle 2, 10, -1 \rangle$

$\bar{x} = \langle x, y, z \rangle \quad \Longrightarrow \quad \bar{n} \cdot (\bar{x} - \bar{p}) = 0$

$\bar{p} = \langle 2, -2, 12 \rangle$

(5) $z = x \sin(x+y)$ @ $(-1,1,0)$

$z_x = \sin(x+y) + x\cos(x+y) \Longrightarrow z_x(-1,1,0) = -1$

$z_y = x\cos(x+y) \Longrightarrow z_y(-1,1,0) = -1$

$\boxed{z = -(x+1) - (y-1)} \Longrightarrow -x-1-y+1 = z$

$x + y + z = 0$

2-03 §14.4 #s 7, 12, 19, 25, 28, 32, 42

#s 7, 8 Graph surface & the indicated tangent plane.

(7) $z = x^2 + xy + 3y^2$ @ $(1,1,5)$

$z_x = 2x + y \Rightarrow z_x(1,1,5) = 3$

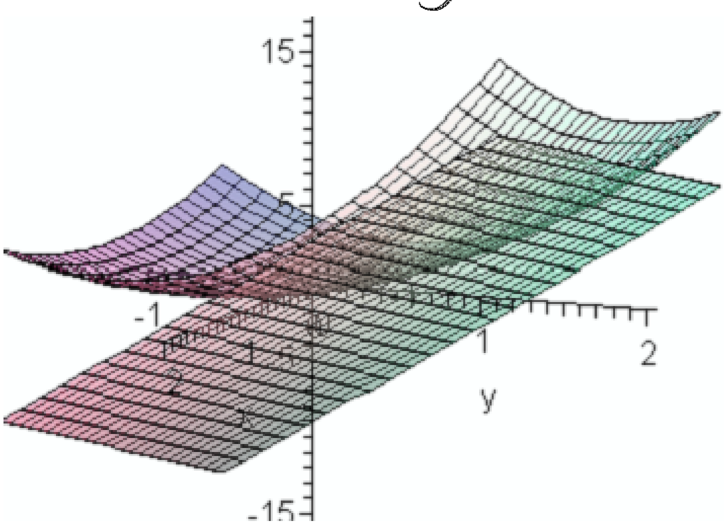
$z_y = x + 6y \Rightarrow z_y(1,1,5) = 7$

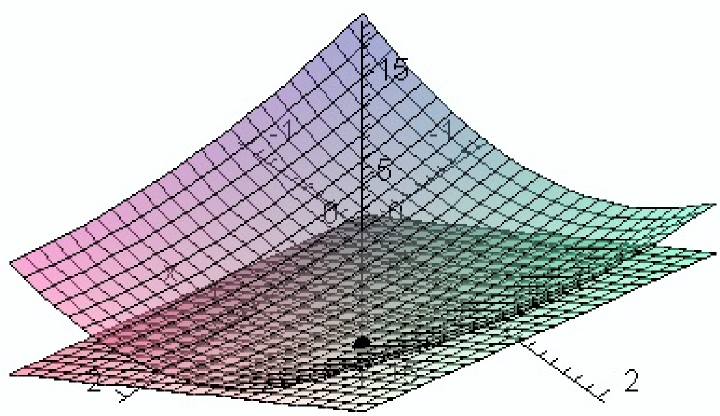
$\Rightarrow z = 3(x-1) + 7(y-1) + 5$

OR $3(x-1) + 7(y-1) - (z-5) = 0$

OR $3x - 3 + 7y - 7 + 5 = z$

$z = 3x + 7y - 5$





203 § 14.4 #'s 12, 19, 25, 28, 32, 42

#'s 11–16 why's it difbl @ the given pt?
Then find linearization at that pt.

(12) $f(x,y) = x^3y^4$ @ $(1,1)$

It's a polynomial; hence, it's difbl
on all of $\mathbb{R}\times\mathbb{R} = \mathbb{R}^2$.

$f(1,1) = 1$

$f_x = 3x^2y^4 \Rightarrow f_x(1,1) = 3$

$f_y = 4x^3y^3 \Rightarrow f_y(1,1) = 4$

$$\boxed{z = 3(x-1) + 4(y-1) + 1}$$

(19) Given f is difbl w/ $f(2,5) = 6$,
$f_x(2,5) = 1$, & $f_y(2,5) = -1$, Estimate
$f(2.2, 4.9)$

$(x_0, y_0, z_0) = (2,5,6)$

$z = 1(x-2) - 1(y-5) + 6$

$z(2.2, 4.9) \approx (2.2-2) - (4.9-5) + 6$

$= .2 + .1 + 6 = \boxed{6.3}$

203 §14.4 #s 25, 28, 32, 42

#s 25-30 Find the differential of the function.

25 $z = e^{-2x}\cos(2\pi t)$

$z_x = -2e^{-2x}\cos(2\pi t)$

$z_y = -2\pi e^{-2x}\sin(2\pi t)$

$$\boxed{dz = -2e^{-2x}\cos(2\pi t)\,dx - 2\pi e^{-2x}\,dy}$$

28 $T = \dfrac{v}{1+uvw} = v(1+uvw)^{-1}$

$T_v = (1+uvw)^{-1} + v(-1)(1+uvw)^{-2}(uw)$

$T_u = -v(1+uvw)^{-2}(vw) = \dfrac{-v^2w}{(1+uvw)^2}$

$T_w = -v(1+uvw)^{-2}(uv) = \dfrac{-uv^2}{(1+uvw)^2}$

$\cancel{dT =}$ $T_v = \dfrac{1+uvw-uvw}{(1+uvw)^2} = \dfrac{1}{(1+uvw)^2}$

$$dT = \boxed{\frac{1}{(1+uvw)^2}\left[dv - v^2w\,du - uv^2\,dw\right]}$$

203 § 14.4 #s 32, 42

32) $z = x^2 - xy + 3y^2$ & (x,y) changes from $(3,-1)$ to $(2.96, -.95)$. Compare dz & Δz

$\Delta z = z(2.96, -.95) - z(3,-1)$

~~$= (2.96)^2 - (2.96)(-.95) - (3^2 - 3(-1) + 3(-1)^2)$~~

~~$= 6.090200 - 6 = \boxed{.090200} = \Delta z$~~

$\Delta x = dx = -.04$

$\Delta y = dy = +.05$

$\boxed{\begin{aligned}\Delta z &= f(2.96, -.95) - f(3,-1) \\ &= -.7189\end{aligned}}$

$dz = (2x - y)dx + (-x + 6y)dy$ @

$(x,y) = (3,-1) \rightarrow$

$dz = (2(3) - (-1))(-.04) + (-3 - 6)(+.05)$

$= 7(-.04) - 9(+.05)$

$= -.28 - .45 = \boxed{-.73}$

203 §14.4 #42

42 $\bar{r}_1(t) = \langle 2+3t, 1-t^2, 3-4t+t^2 \rangle$

$\bar{r}_2(u) = \langle 1+u^2, 2u^3-1, 2u+1 \rangle$

lie on surface S!

Want tan plane @ $P(2,1,3)$

Let $\bar{p} = \langle 2,1,3 \rangle$

$\bar{r}_1(0) = \langle 2,1,3 \rangle$ good

$\bar{r}_2(1) = \langle 2,1,3 \rangle$ sweet!

$\bar{r}_1{}'(t) = \langle 3, -2t, -4+2t \rangle \Longrightarrow$

$\bar{u} = \bar{r}_1{}'(0) = \langle 3, 0, -4 \rangle$ is d-vect

$\bar{r}_2{}'(t) = \langle 2u, 6u^2, 2 \rangle \longrightarrow$

$\bar{v} = \bar{r}_2{}'(1) = \langle 2, 6, 2 \rangle$.

~~Not Quite~~ !

VECTOR EQ'N:

$\bar{x} = s\bar{u} + t\bar{v}$,

$s, t \in \mathbb{R}$

You're missing $\bar{p}$!

$$\begin{array}{ll} \bar{r}_1{}'(0) & \langle 3, 0, -4 \rangle \\ \times\ \bar{r}_2{}'(1) & \times \langle 2, 6, 2 \rangle \\ \hline \end{array}$$

$\bar{n} = \langle 24, -14, 18 \rangle$

OR $\langle 12, -7, 9 \rangle$ works

$\bar{n} \cdot (\bar{x} - \bar{p}) = 0$

$\langle 12, -7, 9 \rangle \cdot \langle x-2, y-1, z-3 \rangle = 0$

$12(x-2) - 7(y-1) + 9(z-3) = 0 \longrightarrow \boxed{12x - 7y + 9z = 44}$

$12x - 24 - 7y + 7 + 9z - 27 = 0$

203 §14.4 #42

Some scribbles on resolving vector eq'n & General form

$\bar{x} = s\bar{u} + t\bar{v}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = s\begin{bmatrix} 3 \\ 0 \\ -4 \end{bmatrix} + t\begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}$$

$3s + 2t = x$

$6t = y \Longrightarrow t = \frac{1}{6}y$

$-4s + 2t = z$

$3s + 2\left(\frac{1}{6}y\right) = x$

$3s + \frac{1}{3}y = x$

$3s = x - \frac{1}{3}y$

$s = \frac{1}{3}x - \frac{1}{9}y$

$3\left(\frac{1}{3}x - \frac{1}{9}y\right) + 2(y) = z$

$x - \frac{1}{3}y + 2y = z$

$x + \frac{5}{3}y = z$

$3x + 5y - 3z = 0$

$$\left[\begin{array}{cc|c} 3 & 2 & x \\ 0 & 6 & y \\ -4 & 2 & z \end{array}\right] \xrightarrow{R1+R3} \left[\begin{array}{cc|c} 3 & 2 & x \\ 0 & 6 & y \\ -1 & 4 & x+z \end{array}\right]$$

$$\begin{array}{c} -R3 \\ R2 \\ R1 \end{array}\left[\begin{array}{cc|c} 1 & -4 & -x-z \\ 0 & 6 & y \\ 3 & 2 & x \end{array}\right] \xrightarrow{-3R1+R3} \left[\begin{array}{cc|c} 1 & -4 & -x-z \\ 0 & 6 & y \\ 0 & 14 & 4x+3z \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -4 & -x-z \\ 0 & 1 & \frac{1}{6}y \\ 0 & 14 & 4x+3z \end{array}\right] \xrightarrow{-14R2+R3} \left[\begin{array}{cc|c} 1 & -4 & -x-z \\ 0 & 1 & \frac{1}{6}y \\ 0 & 0 & -\frac{14}{6}y+4x+3z \end{array}\right]$$

$4x - \frac{7}{3}y + 3z = 0$ Got the proper

$12x - 7y + 9z = 0$ coefficients, but constant term is off, b/c I neglected the $\bar{p}$ in the writing of the plane's eq'n in vector form

OOPS! I forgot

$\bar{x} = \bar{p} + s\bar{u} + t\bar{v}$

Forgot!

$\bar{p} = \langle 2,1,3 \rangle$ = starting point